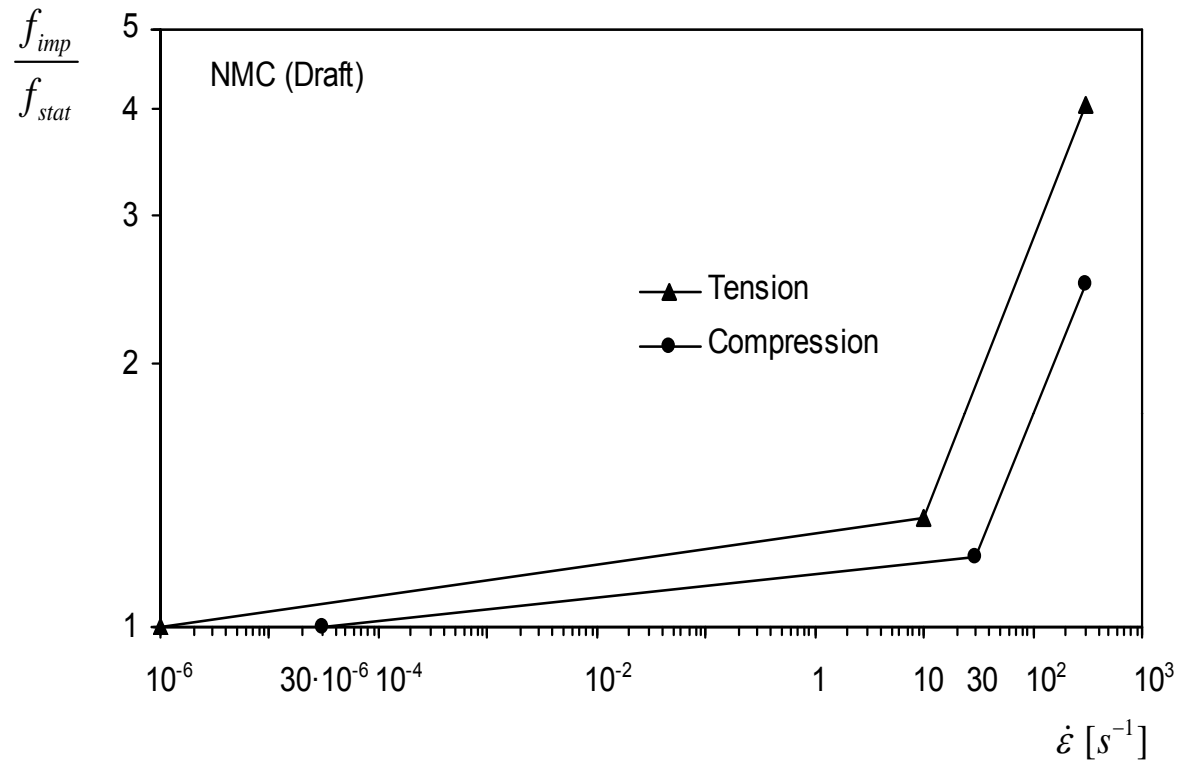


FRACTURE OF CONCRETE AT HIGH LOADING RATES

Introduction



Traffic: $10^{-6} - 10^{-4} s^{-1}$
 Earthquake: $5 \cdot 10^{-3} - 5 \cdot 10^{-1} s^{-1}$
 Airplane impact: $5 \cdot 10^{-2} - 2 \cdot 10^0 s^{-1}$
 Hard impact: $10^0 - 5 \cdot 10^1 s^{-1}$
 Hypervelocity impact: $10^2 - 10^6 s^{-1}$

Problem: solution of 3D wave propagation equation for nonlinear media !

Introduction

Two different aspects of the problem:

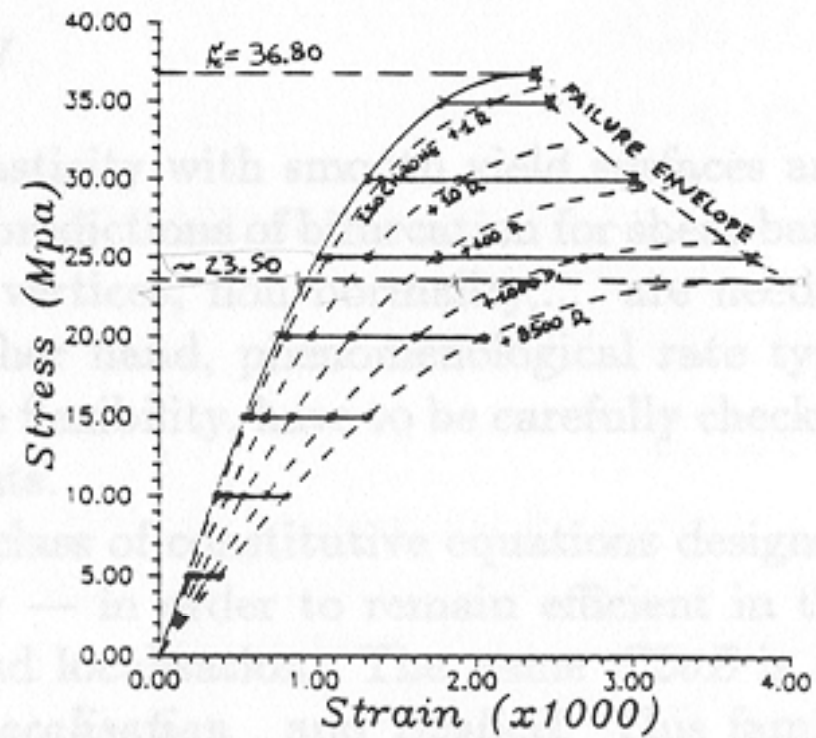
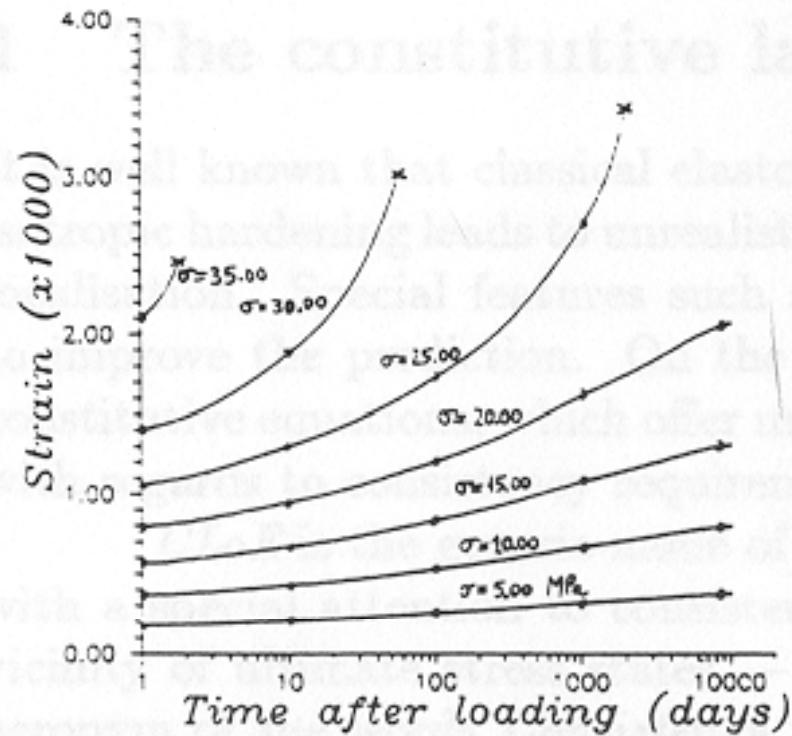
Constitutive law (microstructural phenomena)

- Creep of concrete between cracks - creep-fracture interaction or non (very slow loading: $< 10^{-8} \text{ s}^{-1}$)
- Rate dependent crack growth & viscosity (moderate loading rates: $10^{-5} - 10^1 \text{ s}^{-1}$)

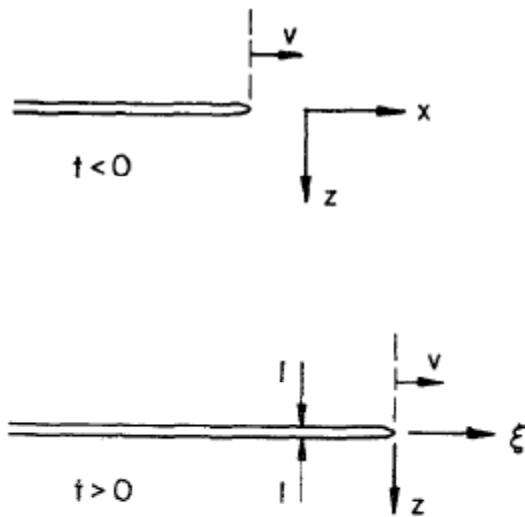
Structural analysis

- Structural inertia (high loading rates – impact: $> 10^1 \text{ s}^{-1}$)
 - Single & multi-body dynamics
 - Contact (collision)

Very slow loading: creep of bulk material

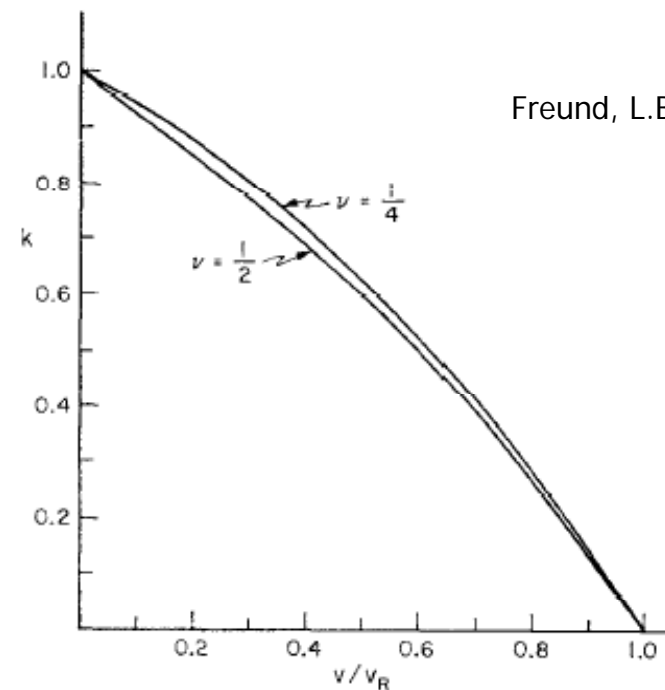


Rate dependent crack growth



$$k = \frac{K_{dynamic}}{K_{static}}$$

Crack propagation in an elastic solid—I.



Freund, L.B. (1971)

Rayleigh wave speed :

$$v_R = \xi(v) \sqrt{\frac{G}{\rho}} \quad \text{for concrete: } \xi(v) \approx 0.91$$

The ratio of dynamic to static stress intensity factors vs. dimensionless crack speed for $\nu = 1/4, 1/2$.

General framework for modeling of rate dependent response

- Continuum mechanics
- Irreversible thermodynamics (isothermal conditions)
- Non-linear fracture mechanics
 - smeared crack approach
 - crack band method
- Basic principles of contact mechanics
- Numerical analysis: standard finite elements

3D static & dynamic finite element analysis

(multi-body + contact)

Global equilibrium – Lagrange multipliers for contact problems:

$$\mathbf{M}\ddot{\mathbf{u}}_n + \mathbf{C}\dot{\mathbf{u}}_n + \mathbf{K}\mathbf{u}_n + \mathbf{G}_{n+1}^T \boldsymbol{\lambda}_n = \mathbf{R}_n$$

Impenetrability: $\mathbf{G}_{n+1} \{\mathbf{x}_{n+1}\} = \mathbf{0}$

Strain measure:	Green-Lagrange strain tensor
Stress tensor:	Co-rotational
FE formulation:	Update Lagrange, mesh refinement, re-meshing
Contact:	Lagrange multipliers, friction
Solution strategy:	Global: explicit FE & Local: implicit contact

Rate dependent crack growth

For continuum with a number of parallel cohesive cracks:

$$\frac{d\varepsilon}{dt} = \frac{\dot{w}}{s_{cr}} + \frac{\dot{\sigma}}{E} \approx \frac{\dot{w}}{s_{cr}}$$

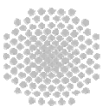
With: ε = average macroscopic strain normal to the crack direction
 s_{cr} = spacing of the parallel cracks
 E = Young's modulus of bulk material

According to the rate process theory

(Krausz and Krausz, 1988; Bažant et al., 2000)

$$\sigma(\dot{\varepsilon}) = \sigma^0(\dot{\varepsilon}) \left[1 + C_2 \ln \left(\frac{2\dot{\varepsilon}}{C_1} \right) \right]$$

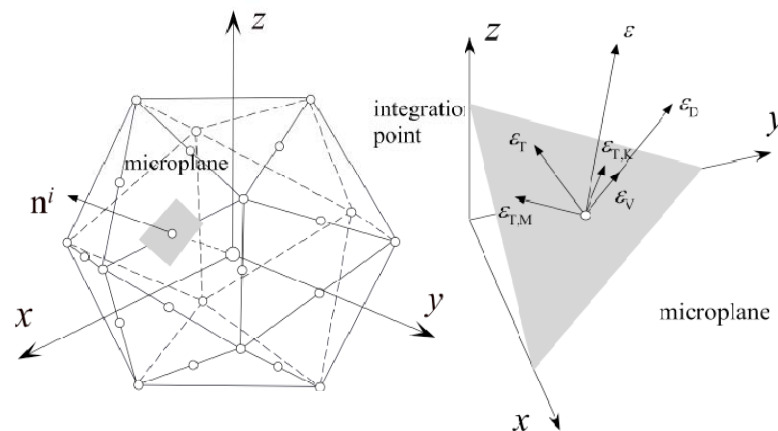
With: σ^0 = rate independent stress; C_1 & C_2 = constants



Rate sensitive microplane model

Microplane model – relaxed kinematic constraint

(Ožbolt et al., 2001)



Kinematic constraint: from $\epsilon_i \rightarrow \epsilon_T, \epsilon_D, \epsilon_N$

$$\sigma_T^p = F_T(\epsilon_T) : \sigma_D^p = F_D(\epsilon_D) : \sigma_N^p = F_N(\epsilon_D, \epsilon_T)$$

Weak form of equilibrium:

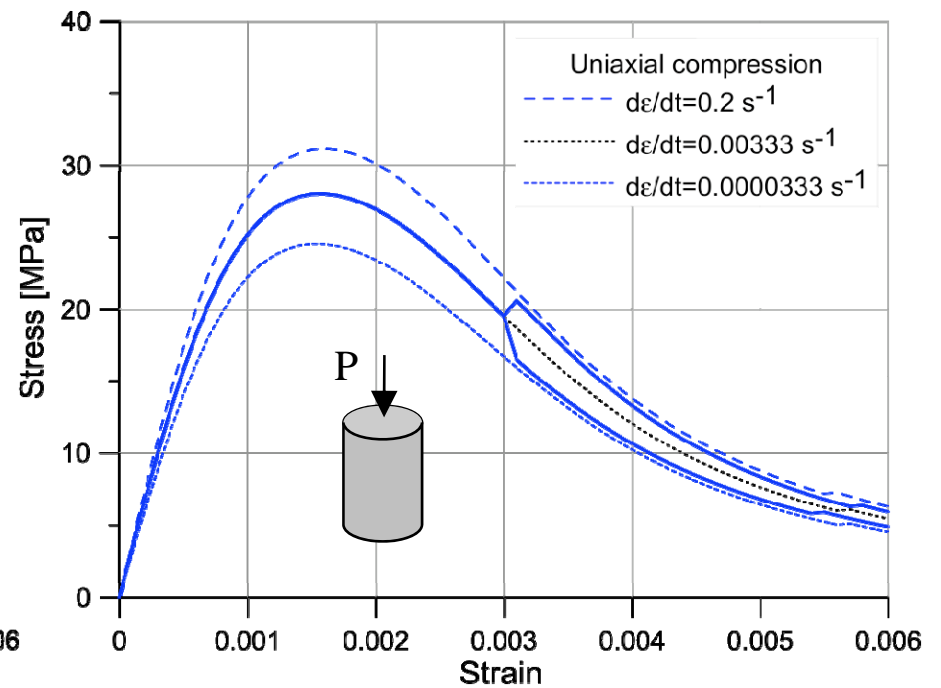
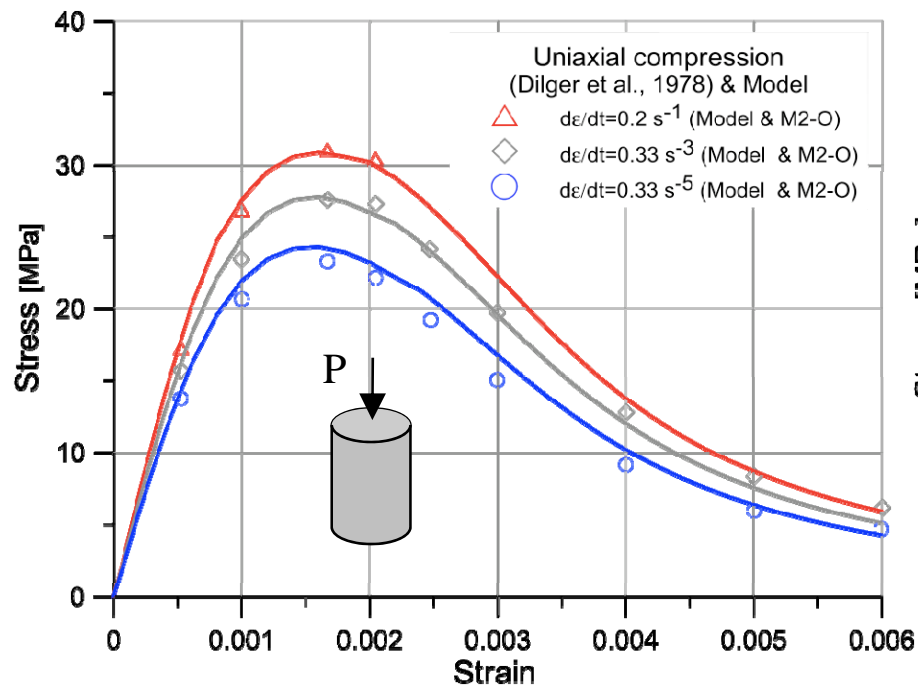
$$\sigma_i^p = \sigma_i^p \delta_i + \frac{3}{2\pi} \int \sigma_D^p (n, n, \frac{\delta_i}{3}) dS + \frac{3}{2\pi} \int \frac{\sigma_N^p}{2} (n, \delta_n \cdot n, \delta_n) dS$$

Influence of the strain rate on the microplane stress components

(Bažant et al., 2000; Ožbolt et al., 2005)

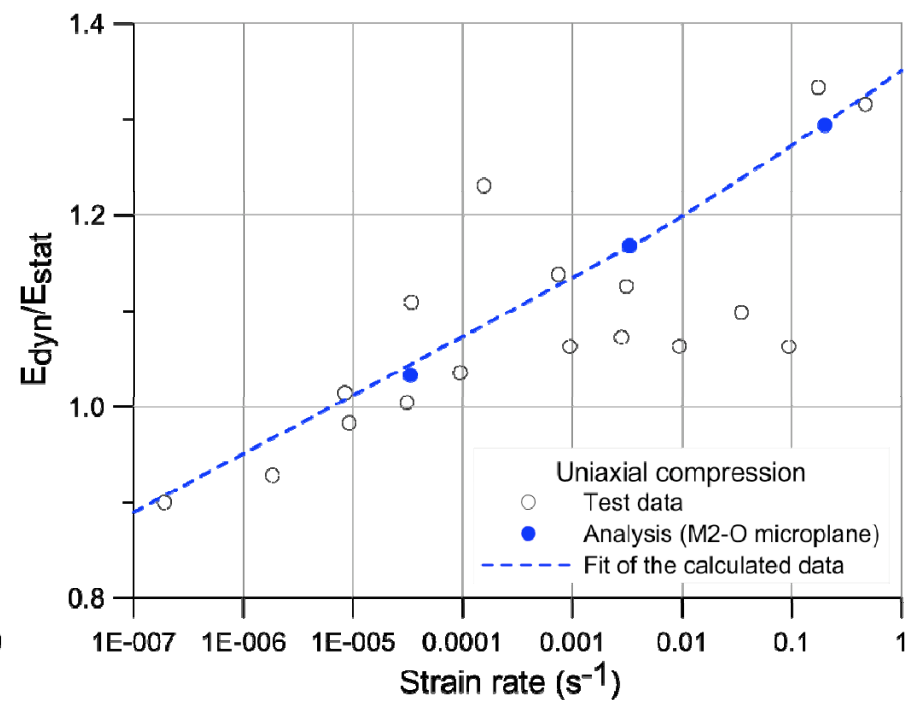
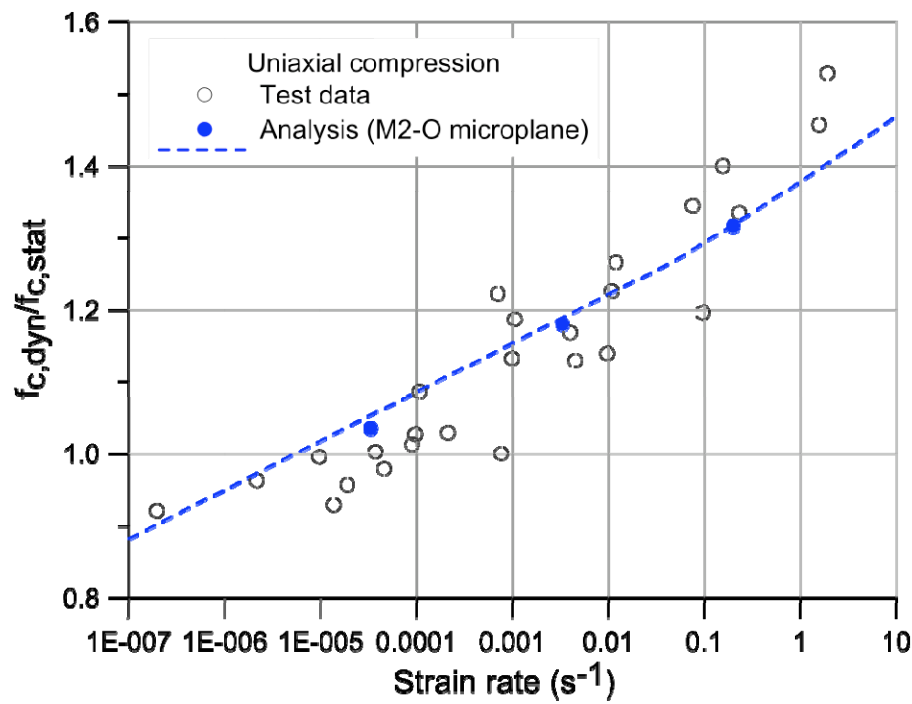
$$\sigma_M(\epsilon_M) = \sigma_M^0(\epsilon_M) \left[1 + c_2 \ln \left(\frac{2\dot{\gamma}}{c_1} \right) \right] \quad \text{with} \quad \dot{\gamma} = \sqrt{\frac{1}{2} \dot{\epsilon}_{ij} \dot{\epsilon}_{ij}} \quad c_1 = \frac{c_0}{s_{cr}}$$

Calibration of the microplane model constitutive level

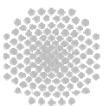


Uniaxial compression (moderate loading rate)

Calibration of the microplane model constitutive level



Uniaxial compression (moderate loading rate)



3D FE analysis of concrete cylinder

Geometry:

$d = 50, 100, 200 \text{ mm}; \quad h = 2d$

Rate independent material properties :

Young's modulus, $E = 30000 \text{ MPa}$

Poisson's ratio, $\nu = 0.18$

$f_t = 2.25 \text{ MPa}$

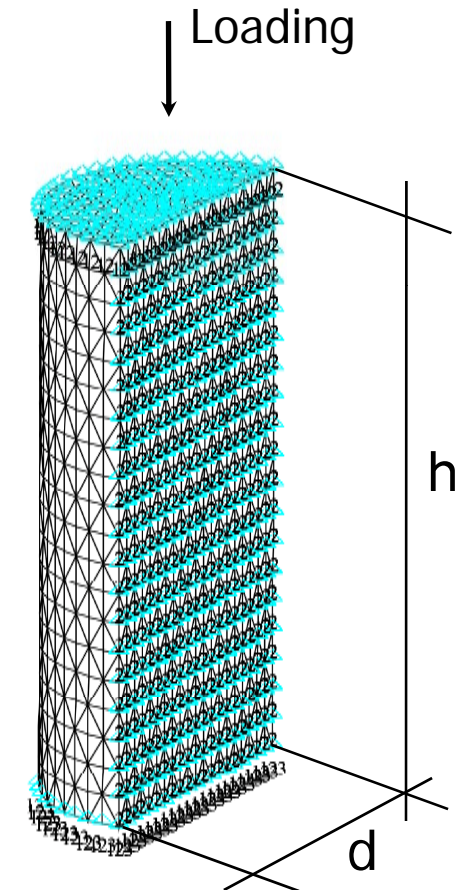
$f_c = 23.0 \text{ MPa}$

$G_F = 0.08 \text{ N/mm}$

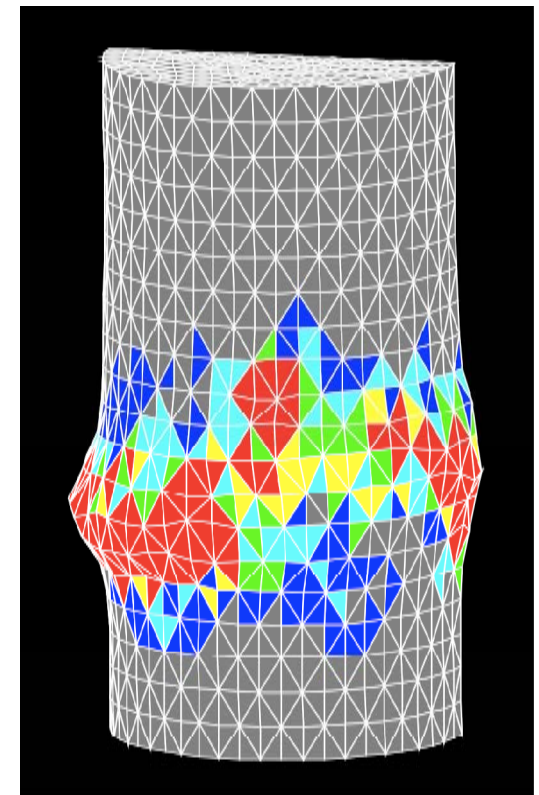
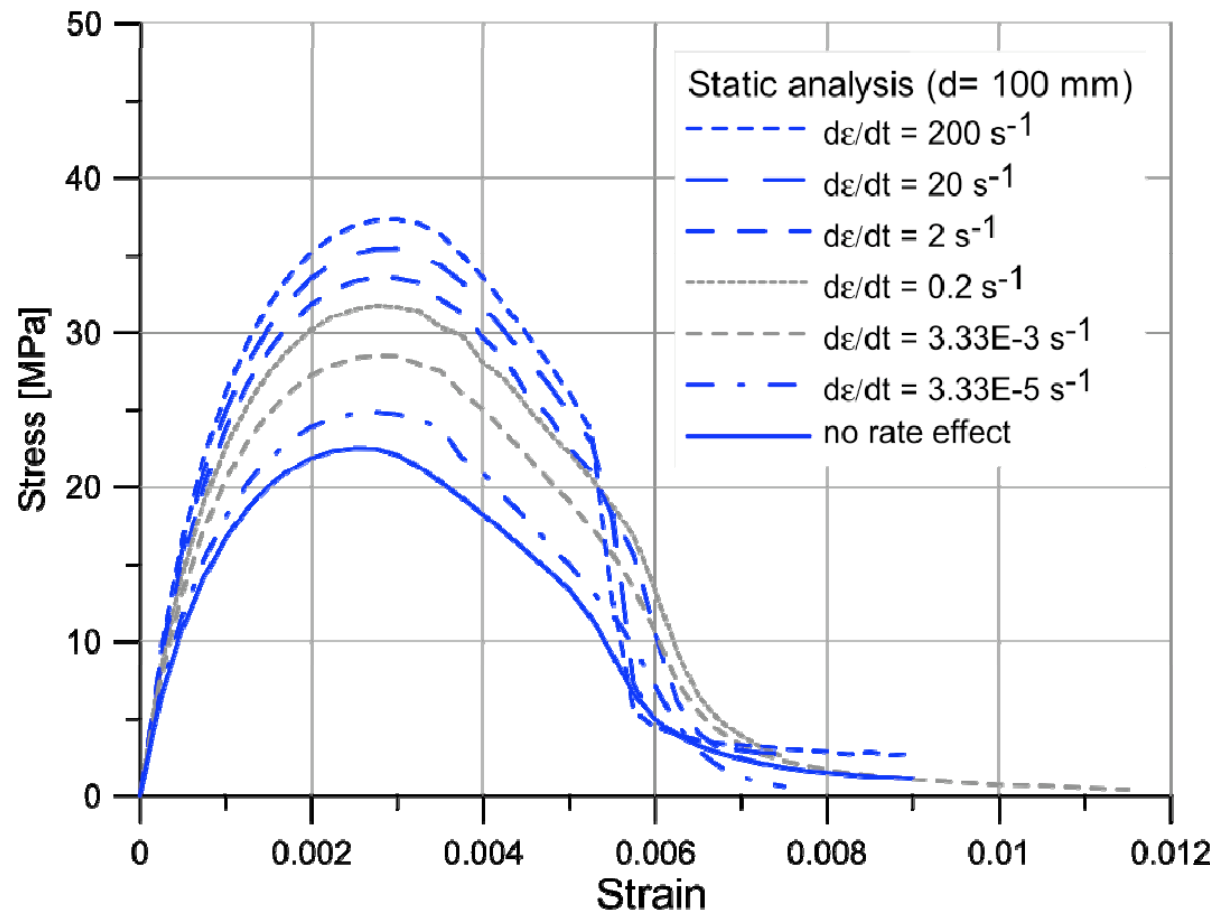
Mass density, $\rho = 2.3 \text{ T/m}^3$

Loading rates $d\varepsilon/dt$ (displacement control):

0 (no rate effect), 3.33^{-5} , 3.33^{-3} , 0.2, 2, 20, 200 s^{-1}

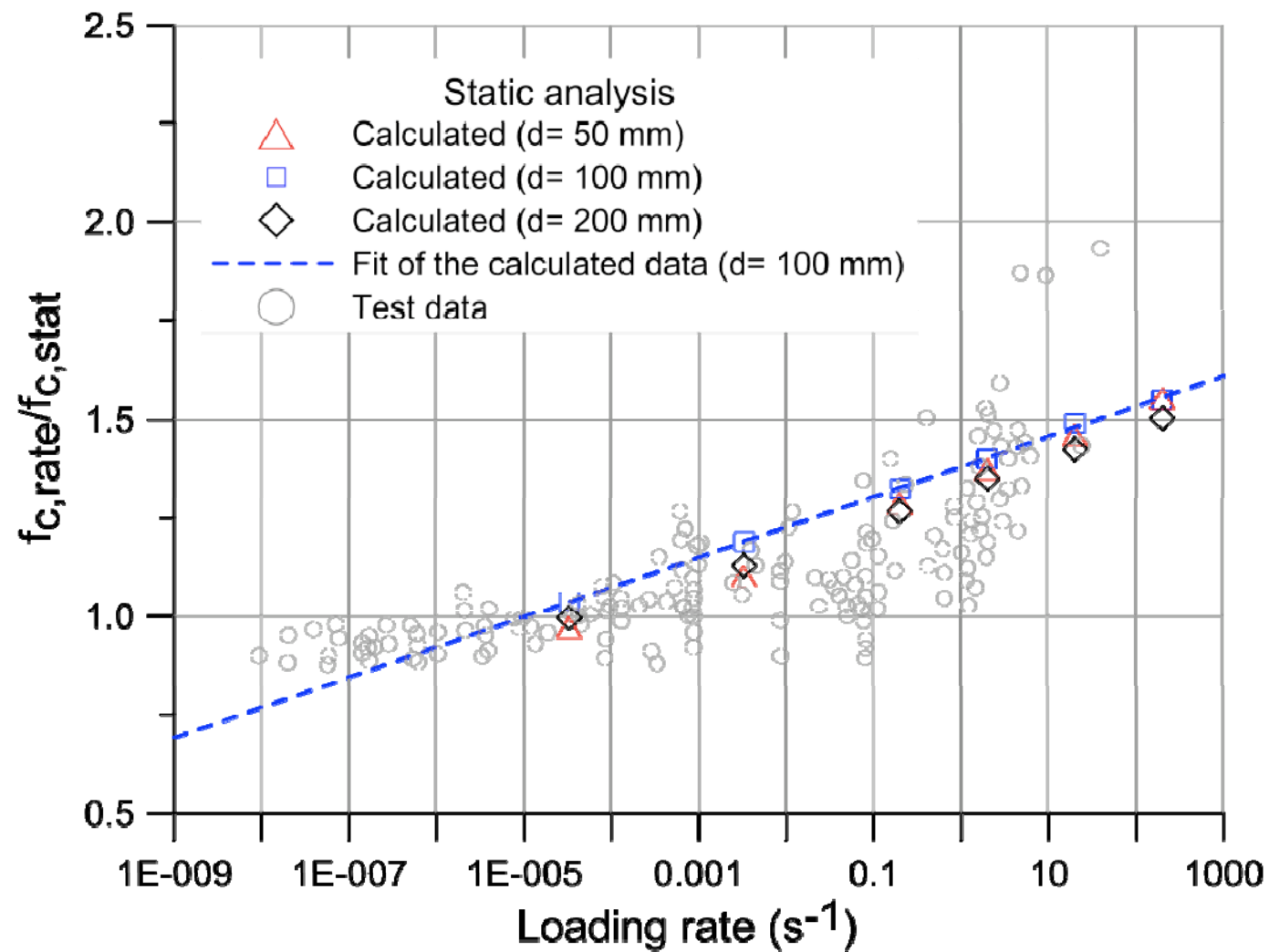


Static analysis



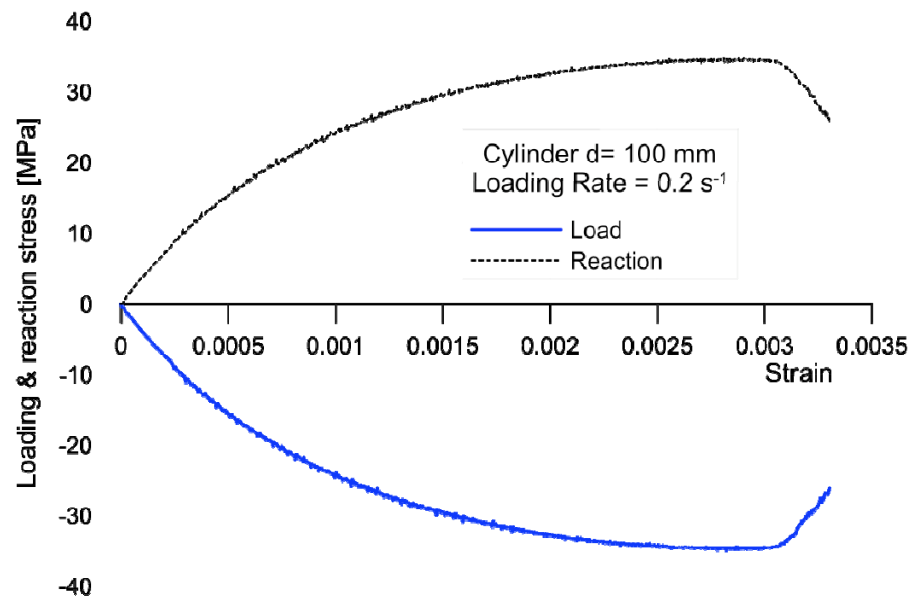
Typical failure mode

Static analysis

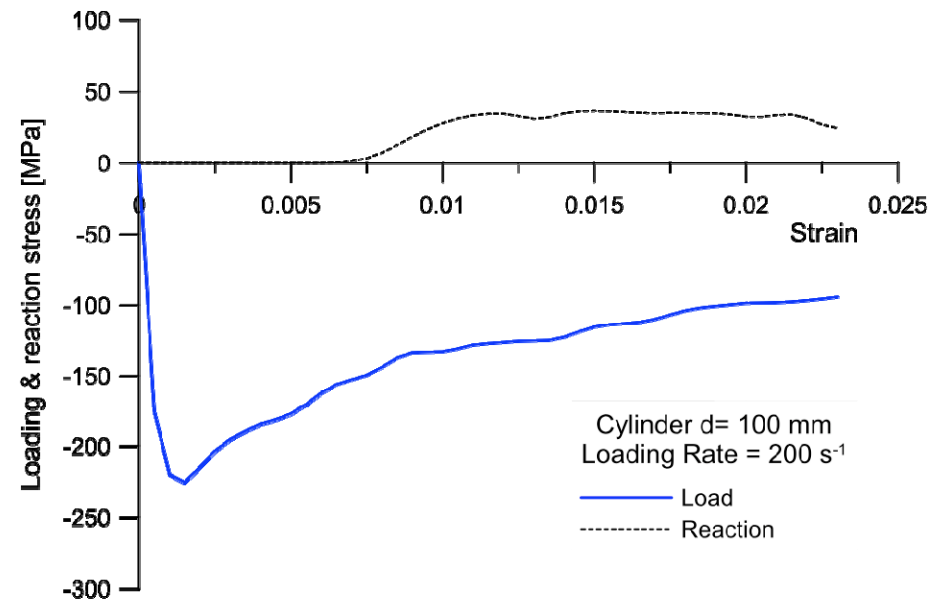


Typical load (reaction) stress-strain curves

static



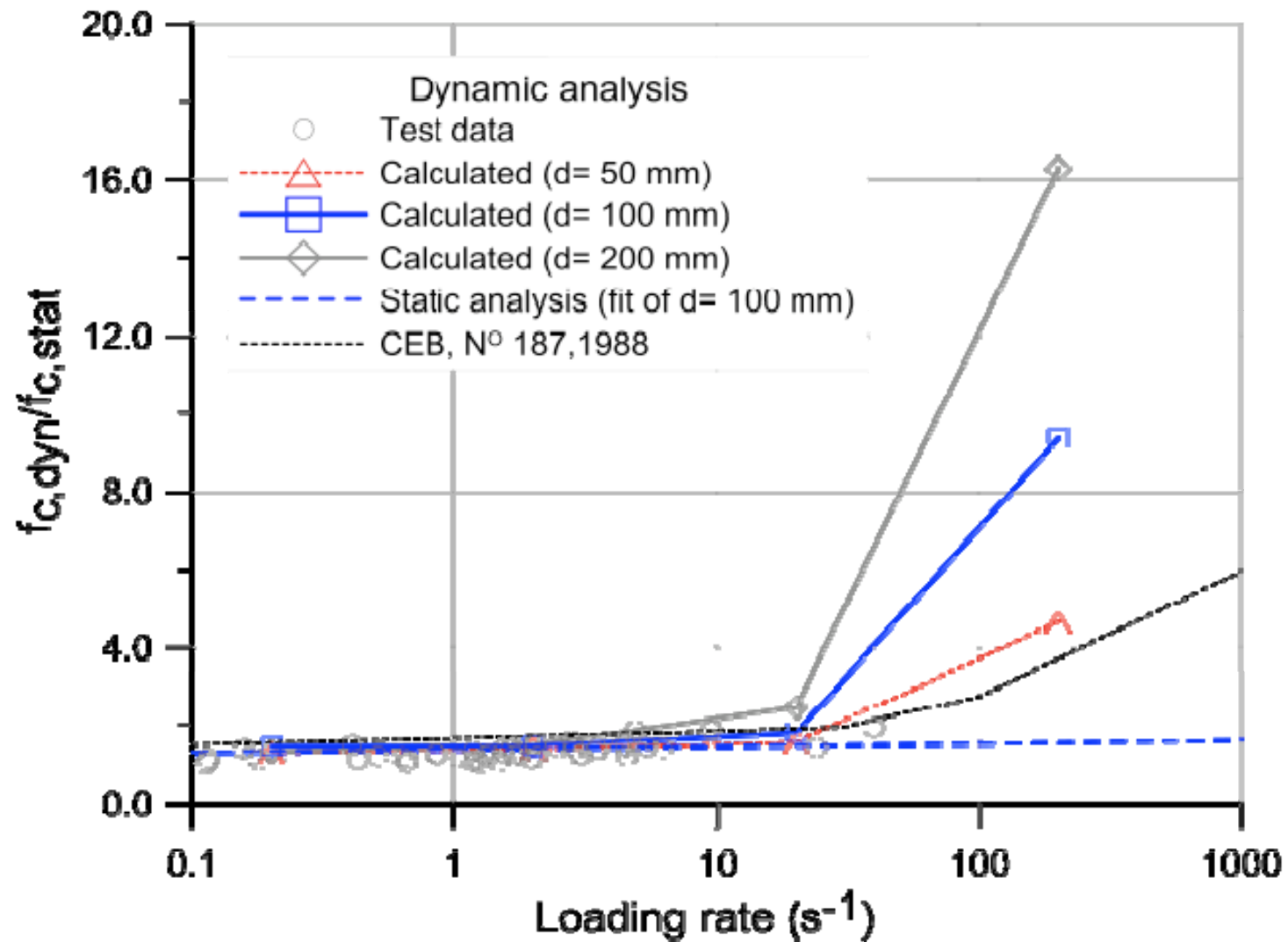
dynamic



d [mm]	0.2 s ⁻¹	2 s ⁻¹	20 s ⁻¹	200 s ⁻¹
50	31.6	33.5	37.7	112
100	35.0	35.2	43.3	225
200	32.0	35.8	59.4	392

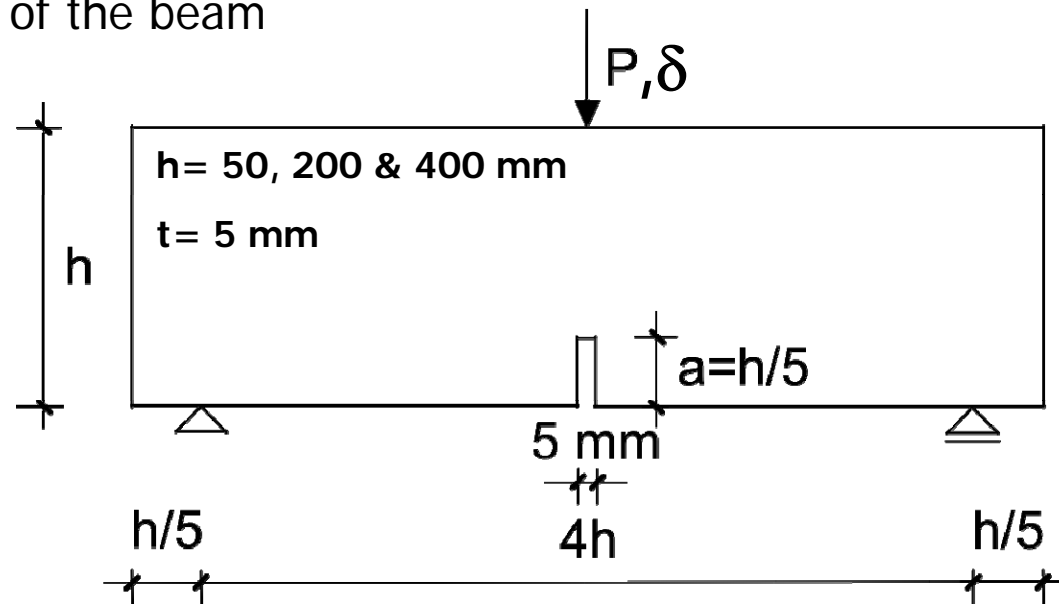
Dynamic analysis - summary of the calculated compressive strengths [MPa]

Strain-rate dependent compressive strength of concrete



Three-point bending – notched beam

Geometry of the beam



Rate independent material properties :

Young's modulus, $E_c = 30000 \text{ MPa}$

Poisson's ratio, $\nu_c = 0.18$

$f_t = 2.0 \text{ MPa}$; $f_c = 30.0 \text{ MPa}$; $G_F = 0.10 \text{ N/mm}$

Mass density, $\rho_c = 2.3 \text{ T/m}^3$

Loading rates ($\Delta\delta/\Delta t$):

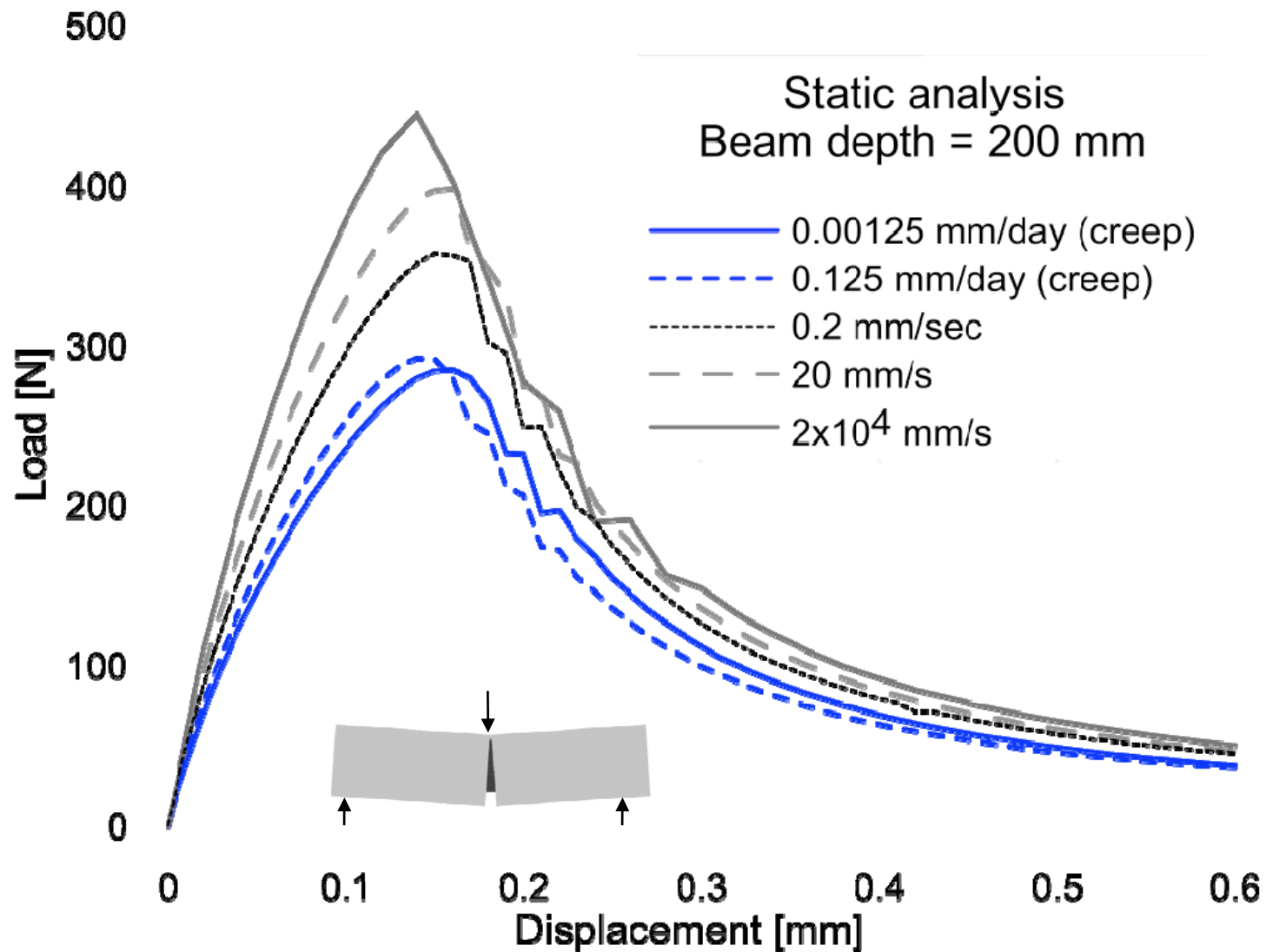
With creep:

1.25×10^{-3} ; 1.25×10^{-2} ; $1.25 \times 10^{-1} \text{ mm/day}$

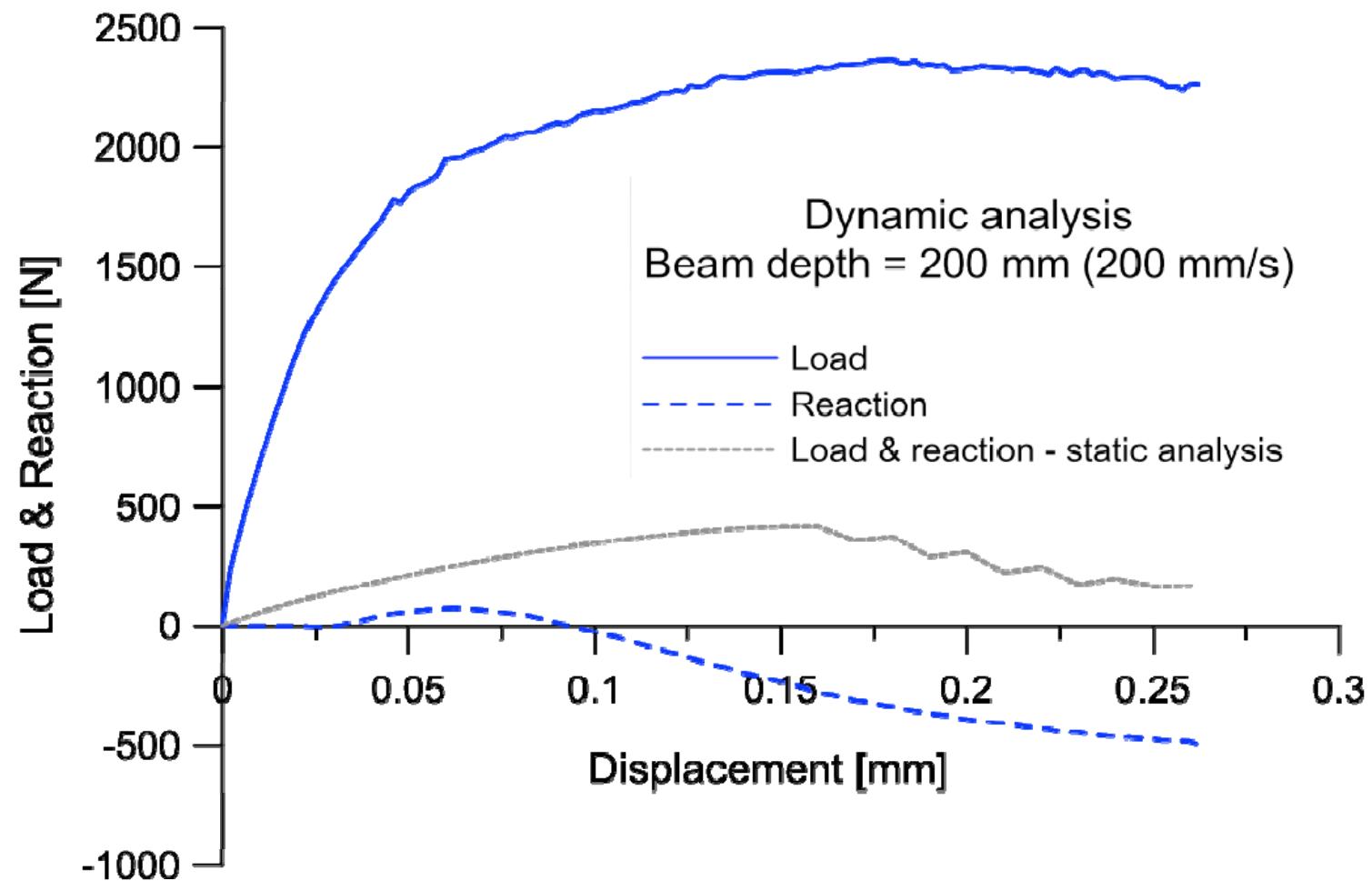
Without creep:

$0.20 - 2.0 \times 10^4 \text{ mm/sec}$

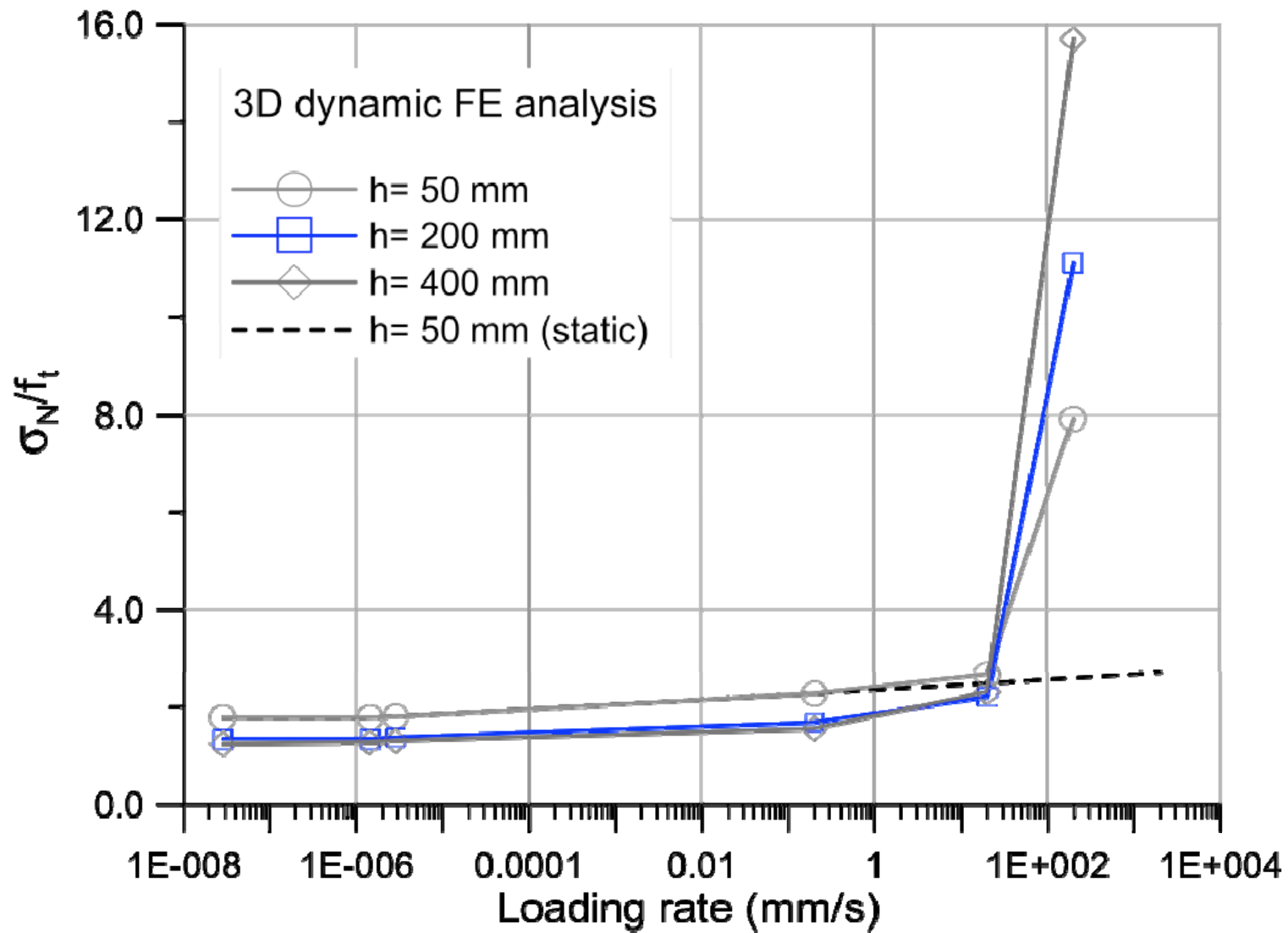
Three-point bending



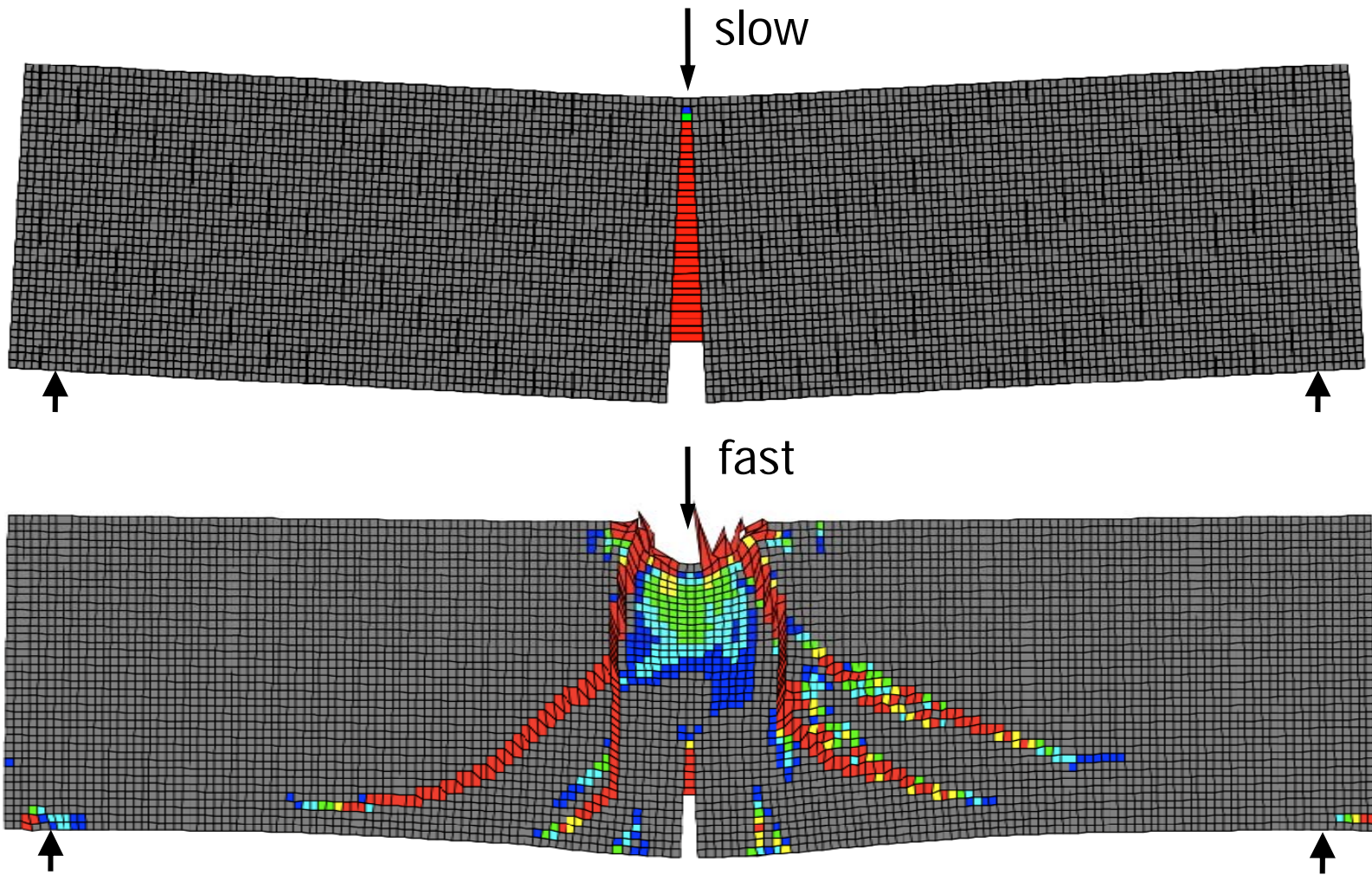
Three-point bending



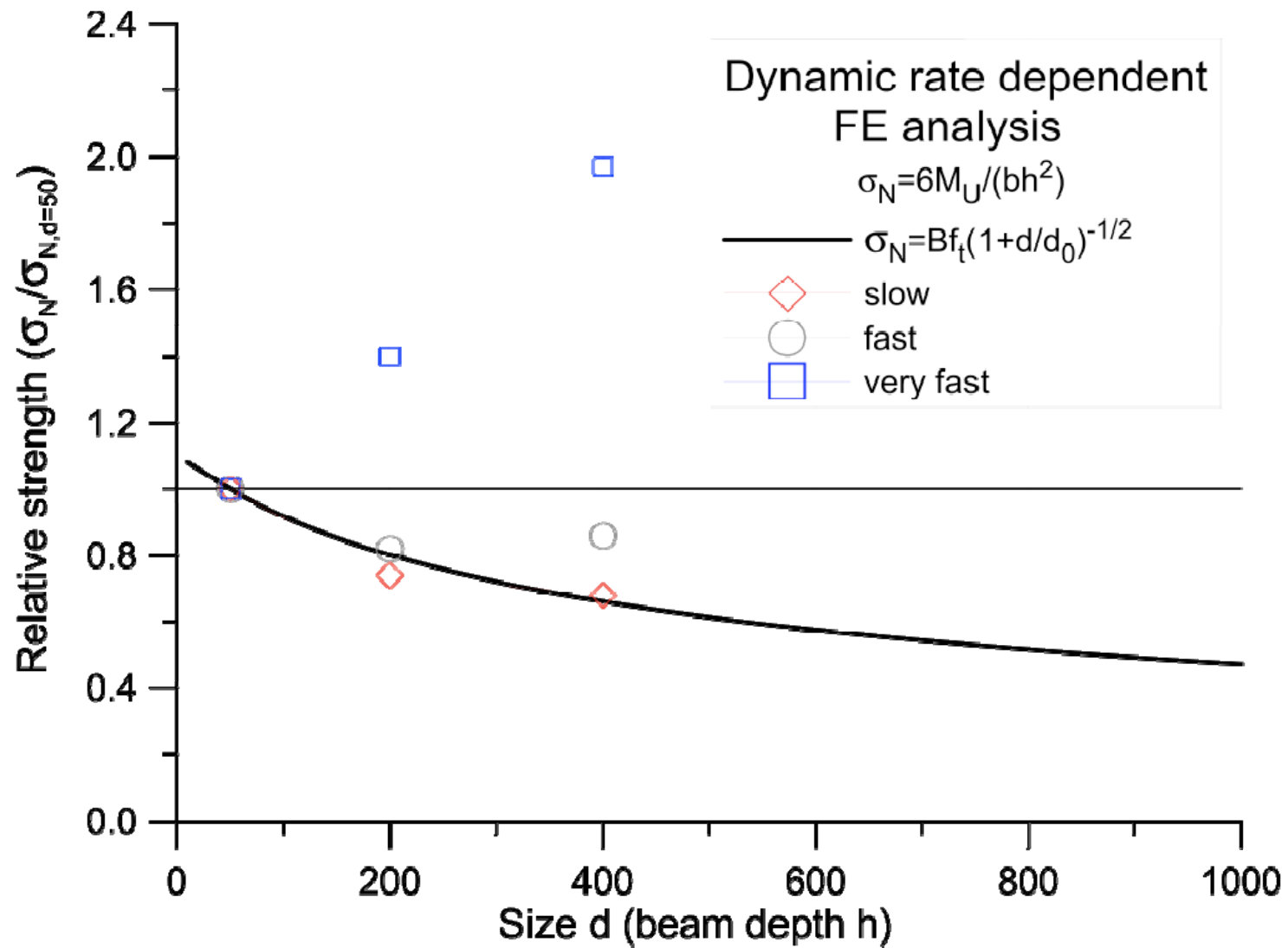
Three-point bending



Three-point bending



Three-point bending



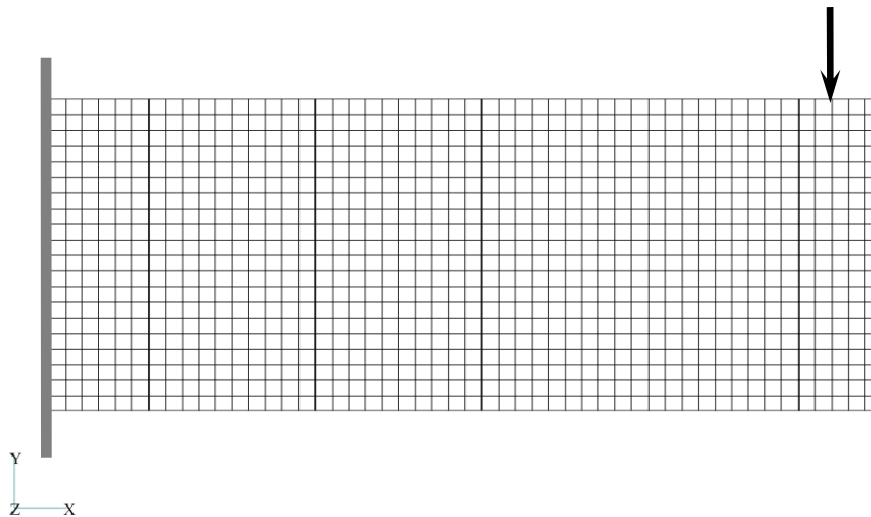
3D FE analysis of cantilever beam

Geometry

$L = 500 \text{ mm}$

$H = 200 \text{ mm}$

$D = 10 \text{ mm}$

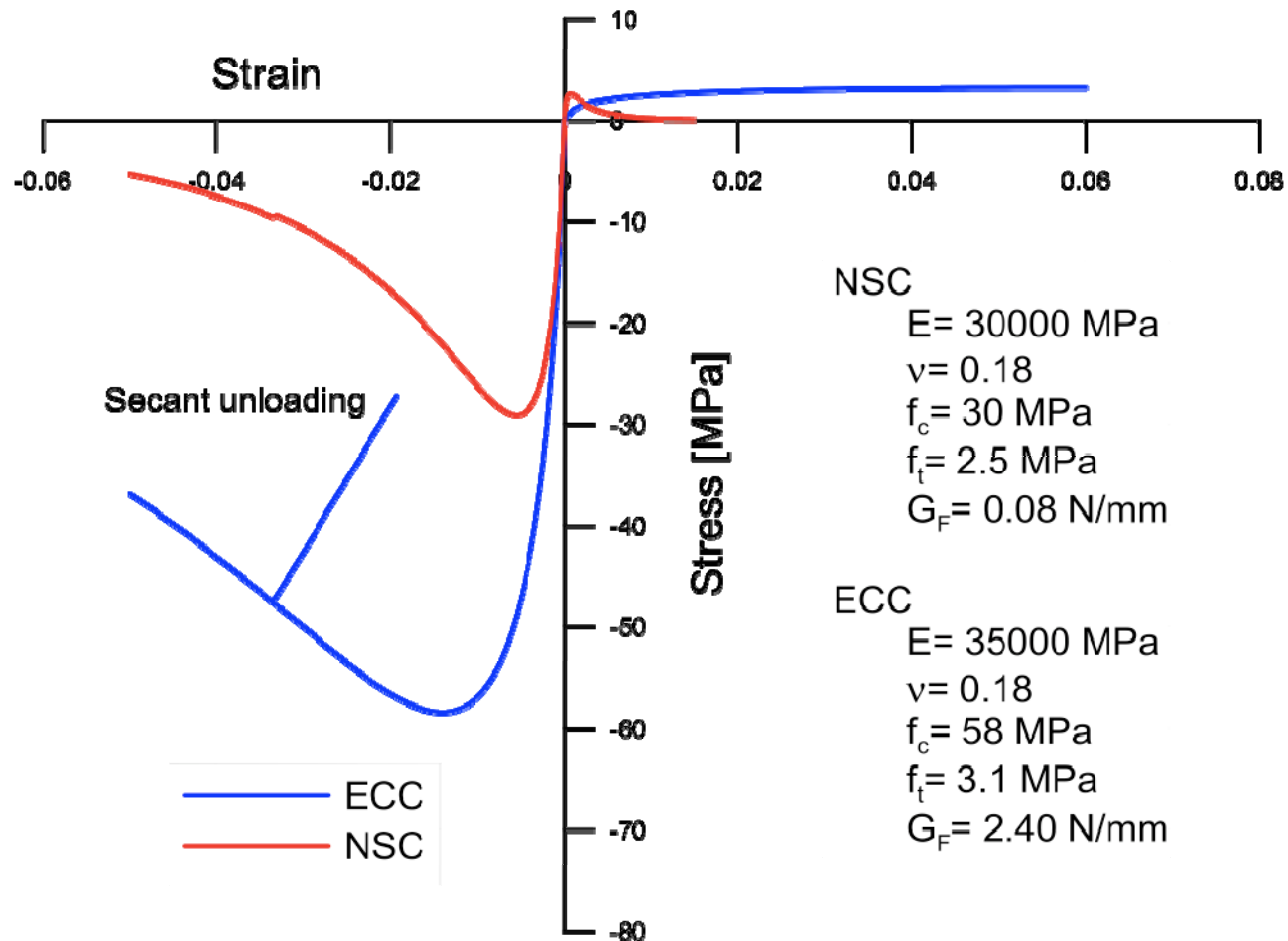


Loading rates (displacement control):

0 (no rate effect), 2, 20, 200, $2 \cdot 10^3$, $2 \cdot 10^4 \text{ mms}^{-1}$

Material properties

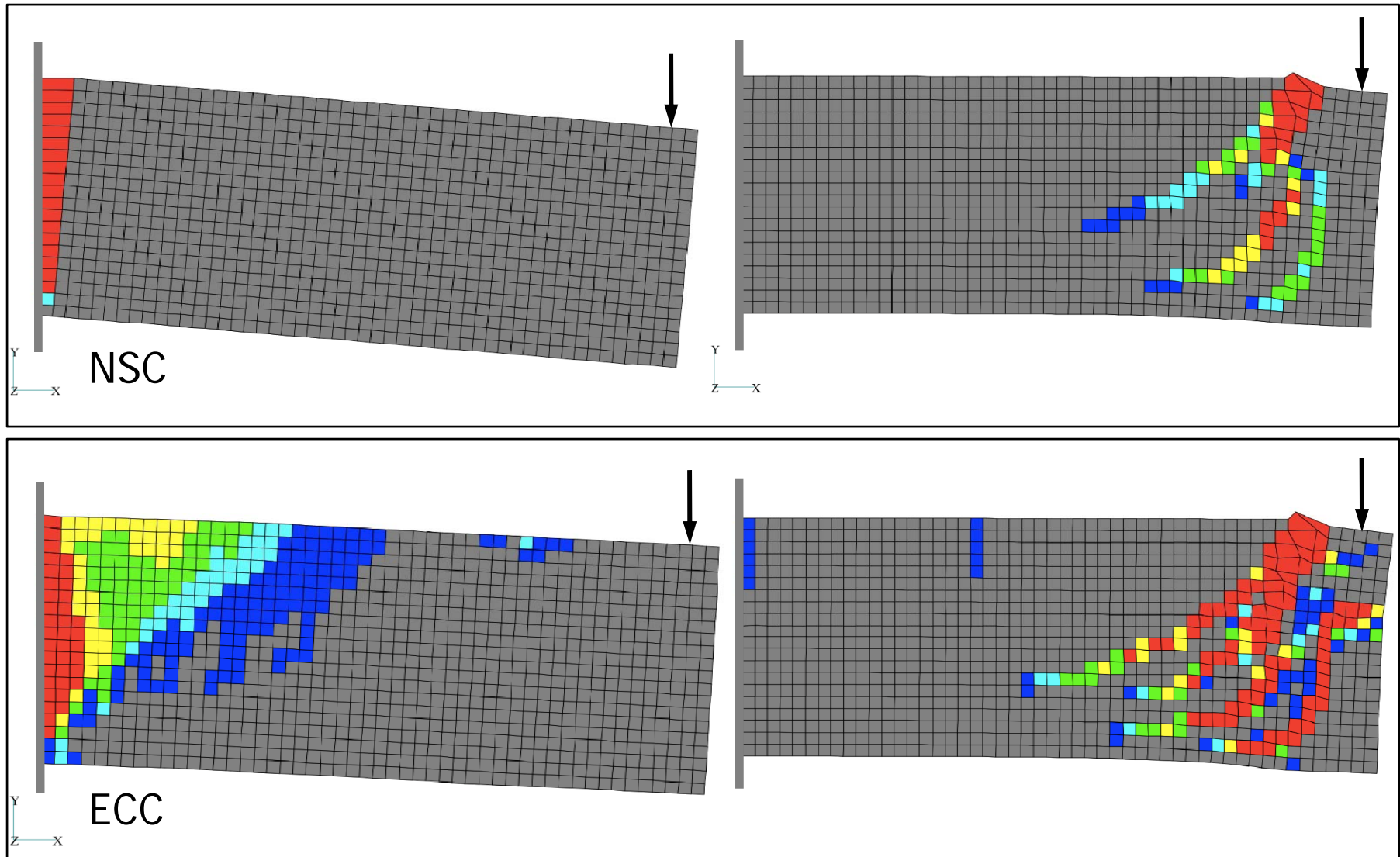
Normal strength concrete (NSC) & high strength fiber reinforced concrete (ECC)



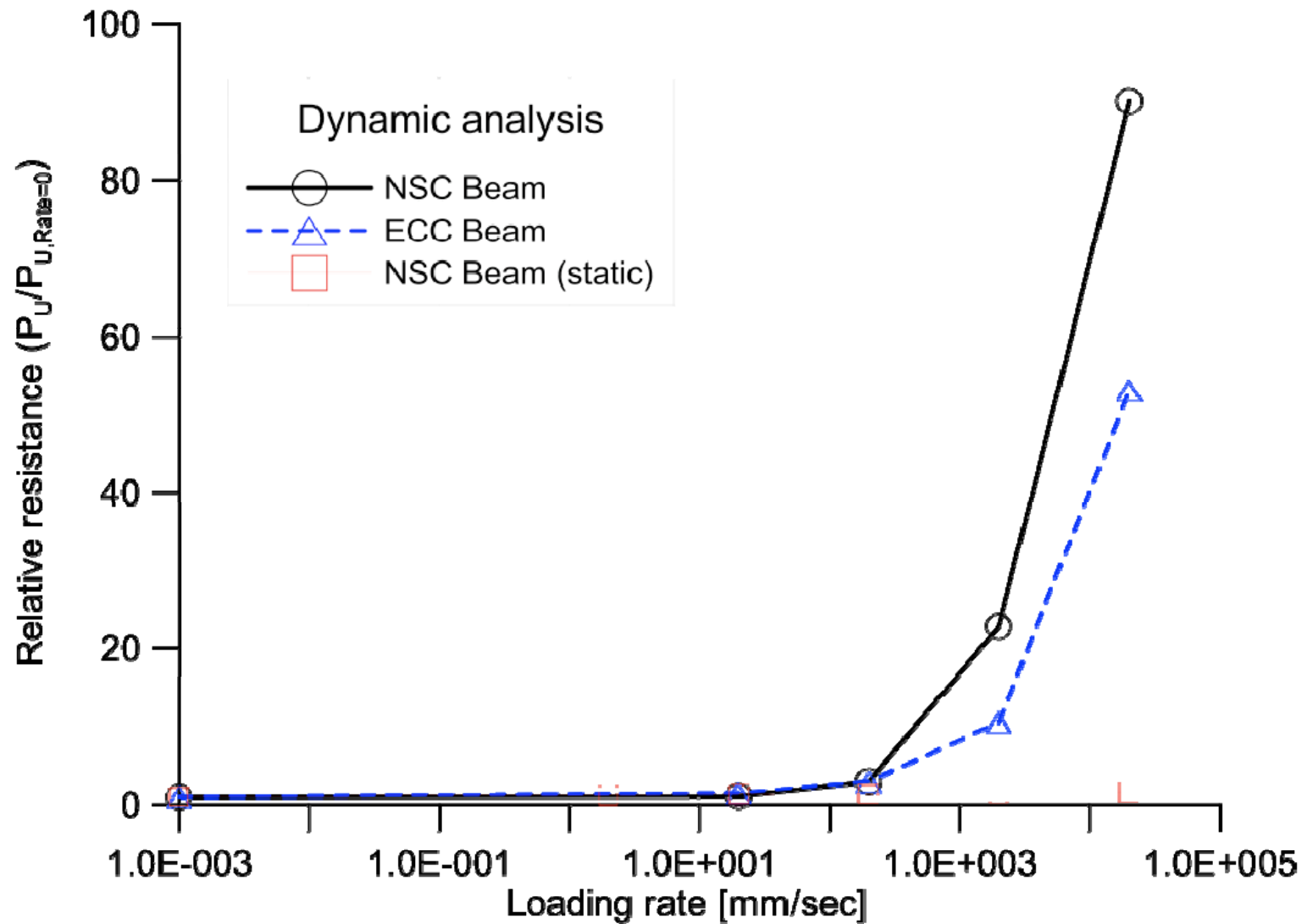
Rate dependent failure modes

slow

fast

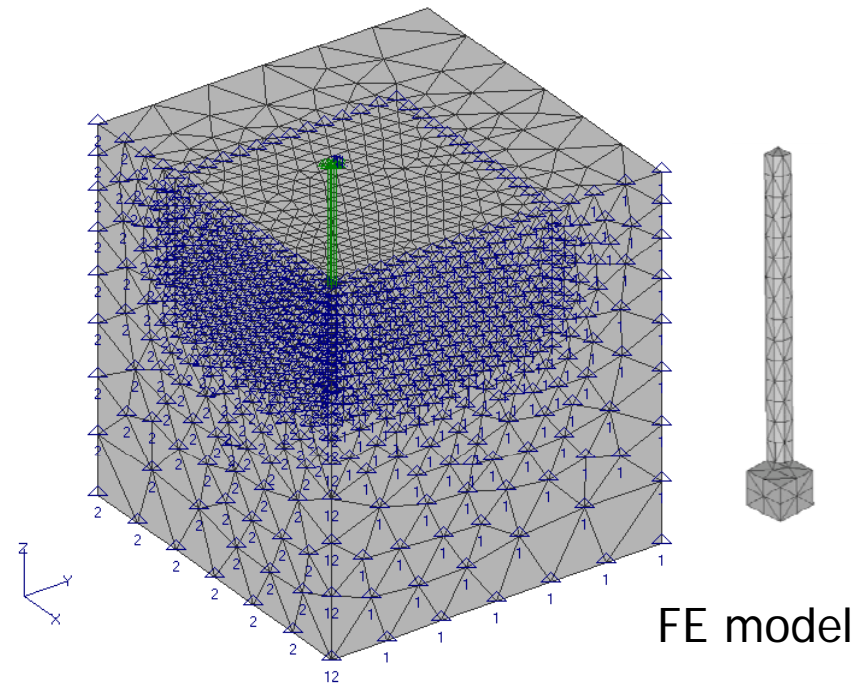
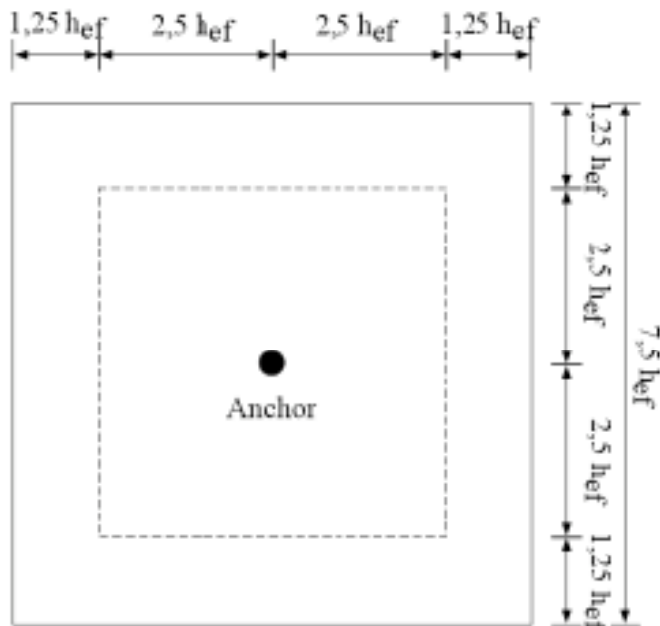
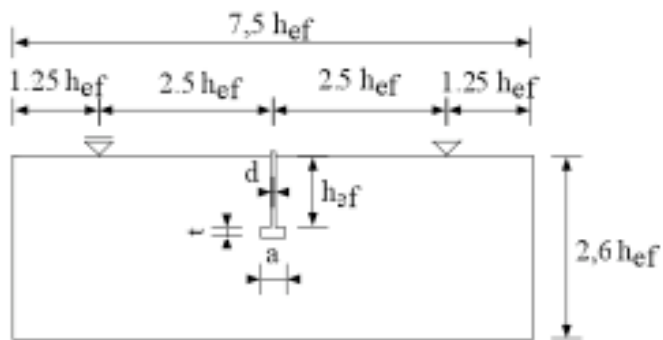


Cantilever beam (NSC & ECC)



Pull-out of the headed stud anchor from a concrete block

Geometry



h_{ef} [mm]	d	t	a
150	16	17	40
889	95.3	102	216
1500	160.8	169	311

Rate independent material properties :

Young's modulus, $E_c = 30000 \text{ MPa}$; $E_s = 200000 \text{ MPa}$

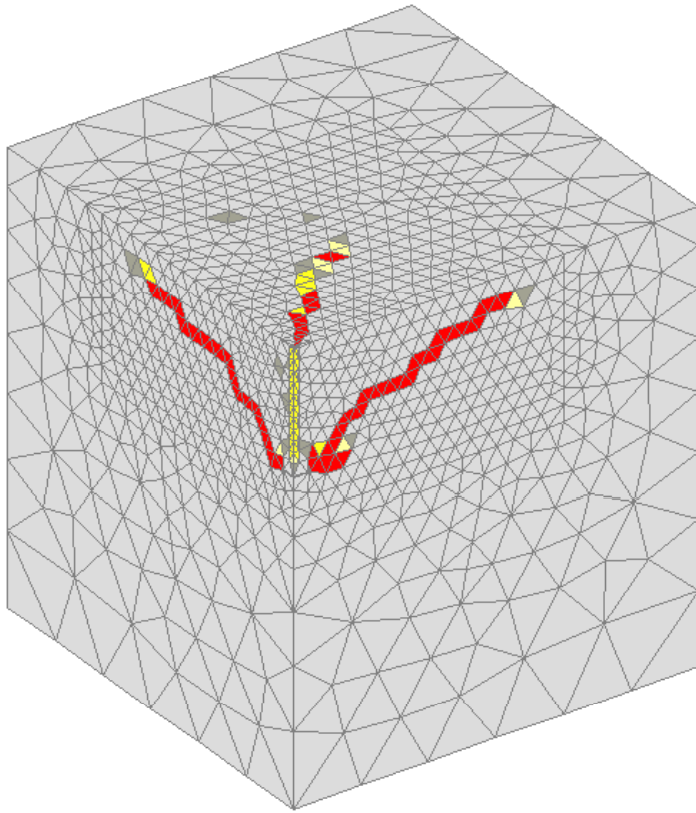
Poisson's ratio, $\nu_c = 0.18$; $\nu_s = 0.33$

$f_t = 2.25 \text{ MPa}$; $f_c = 23.0 \text{ MPa}$; $G_F = 0.08 \text{ N/mm}$

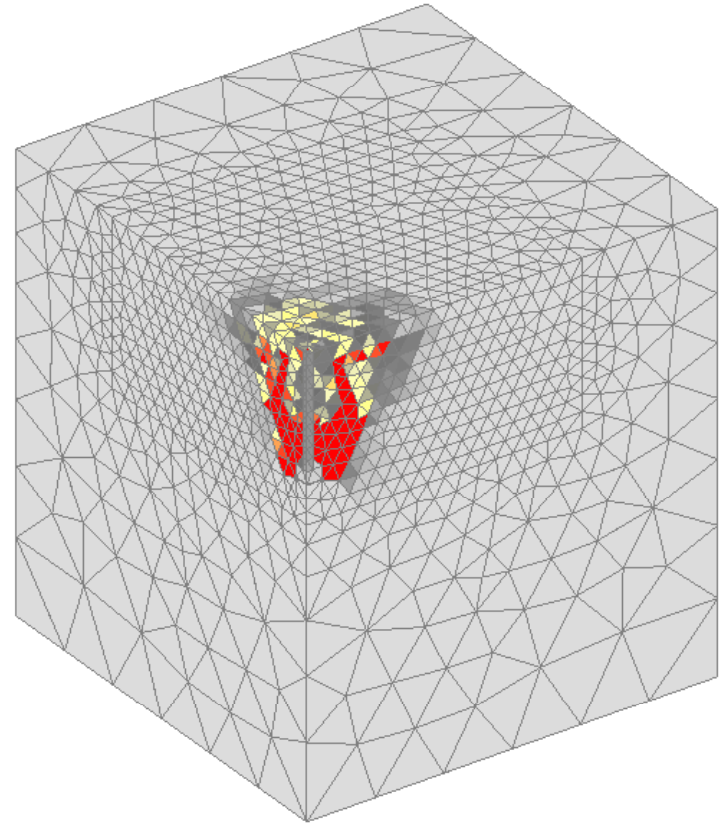
Mass density, $\rho_c = 2.3 \text{ T/m}^3$; $\rho_s = 7.4 \text{ T/m}^3$

Steel – linear elastic

Rate dependent failure modes ($h_{ef} = 150$ mm)

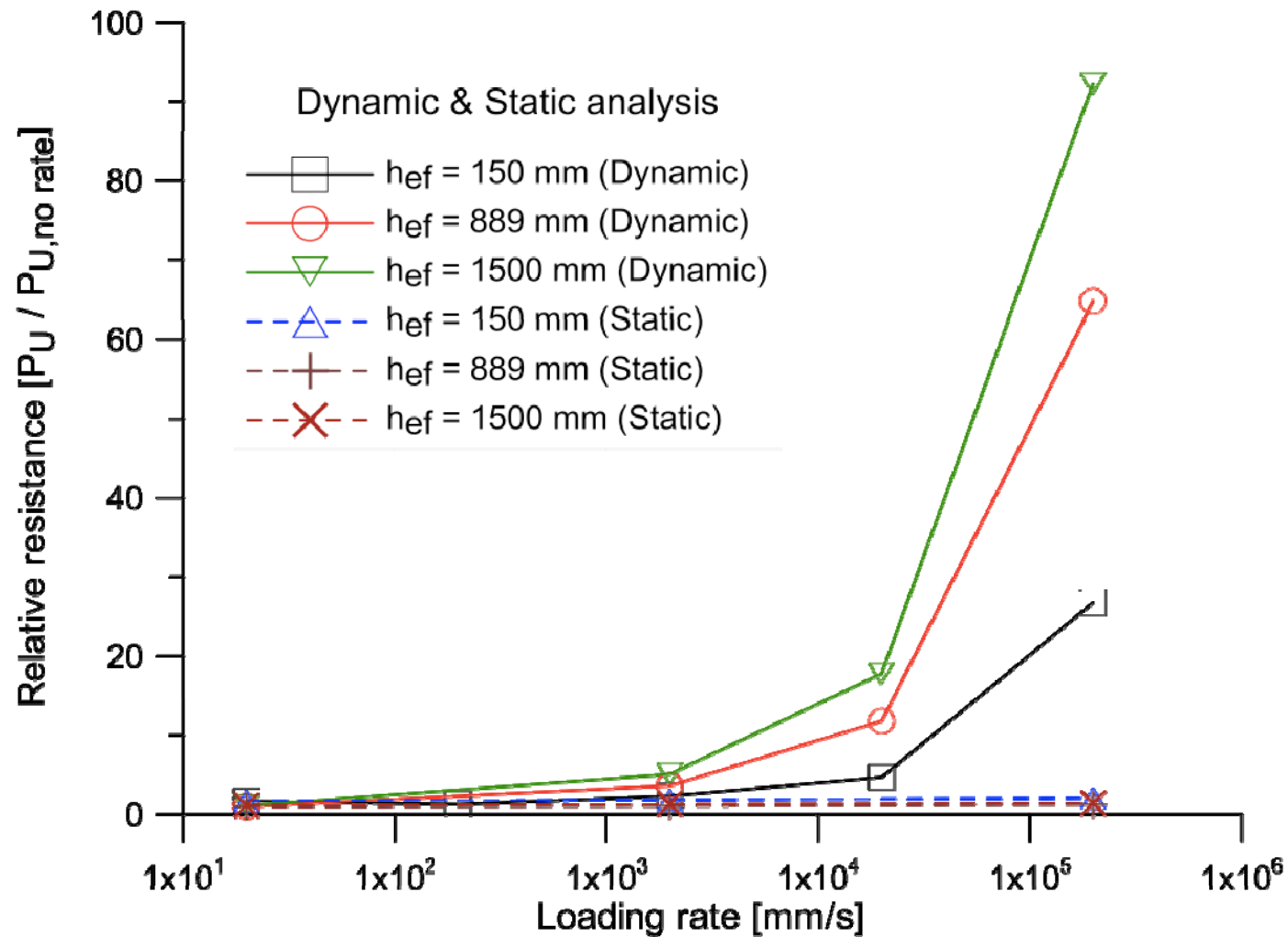


Moderately fast loading (200 mms^{-1})

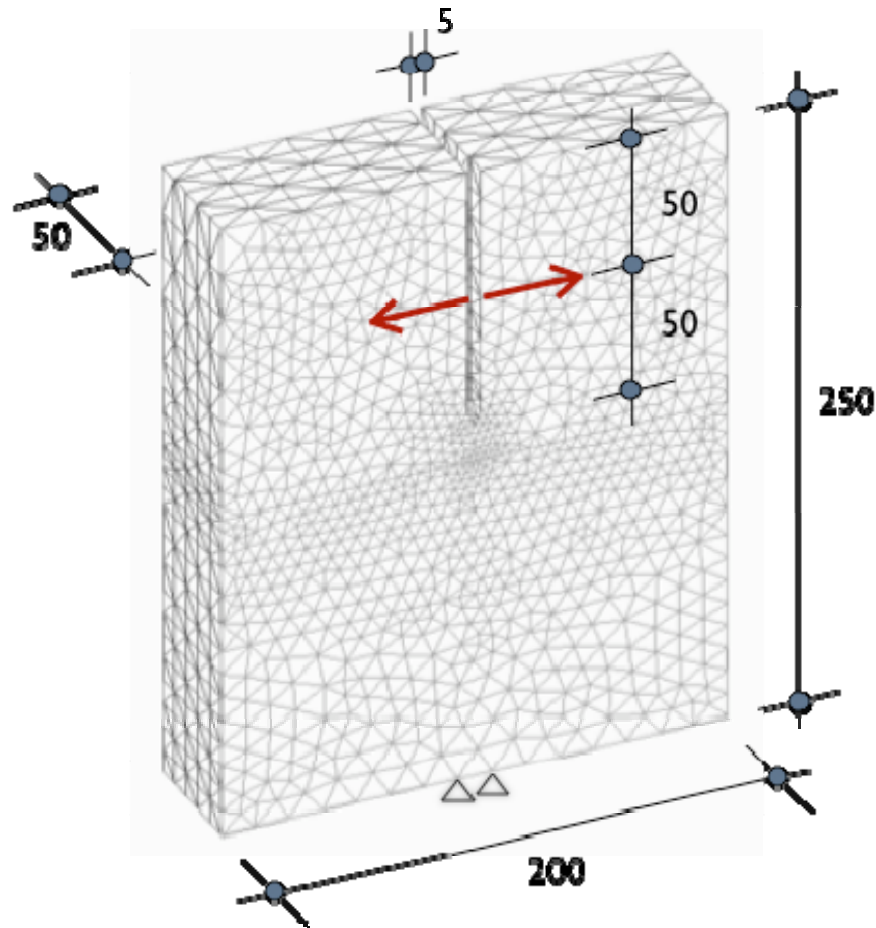


Very fast loading ($2 \cdot 10^5 \text{ mms}^{-1}$)

Pull-out of the headed stud anchor from a concrete block



Compact tension specimen



Rate independent material properties :

Young's modulus, $E_c = 30000$ MPa

Poisson's ratio, $\nu_c = 0.18$

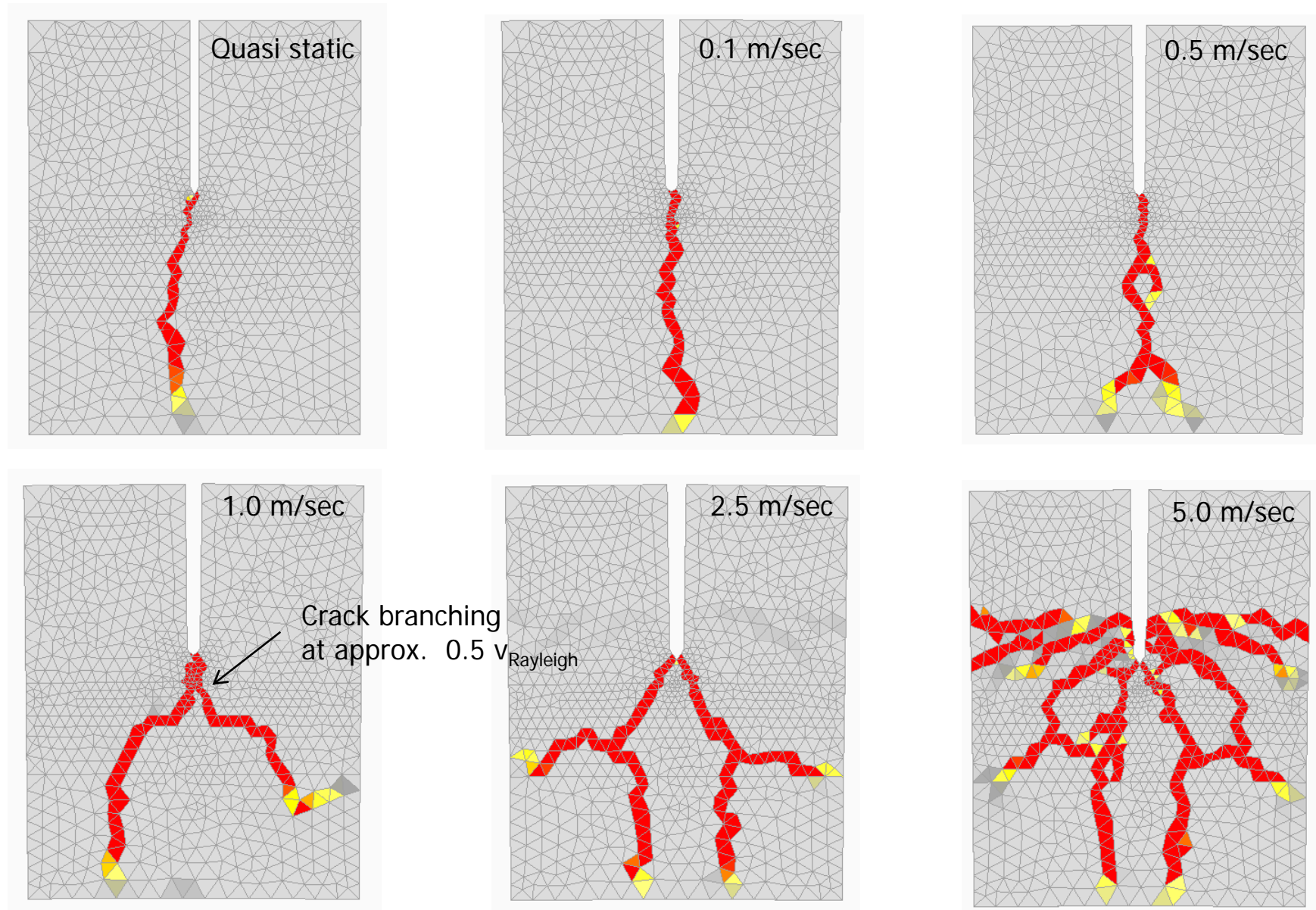
$f_t = 3.5$ MPa; $f_c = 40.0$ MPa; $G_F = 0.08$ N/mm

Mass density, $\rho_c = 2.4$ T/m³

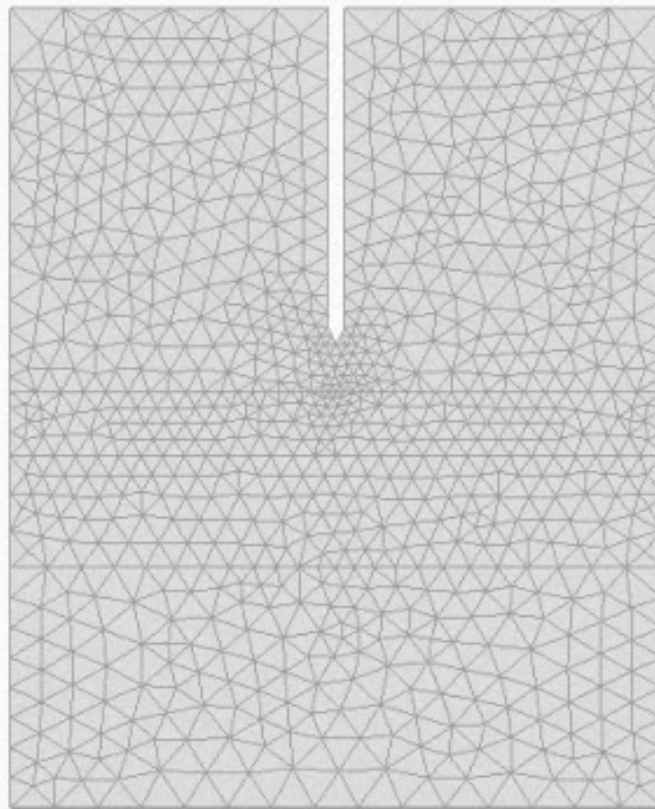
Loading rates ($\Delta/\Delta t$):

Quasi static, 100, 1000 & 1000 mm/sec

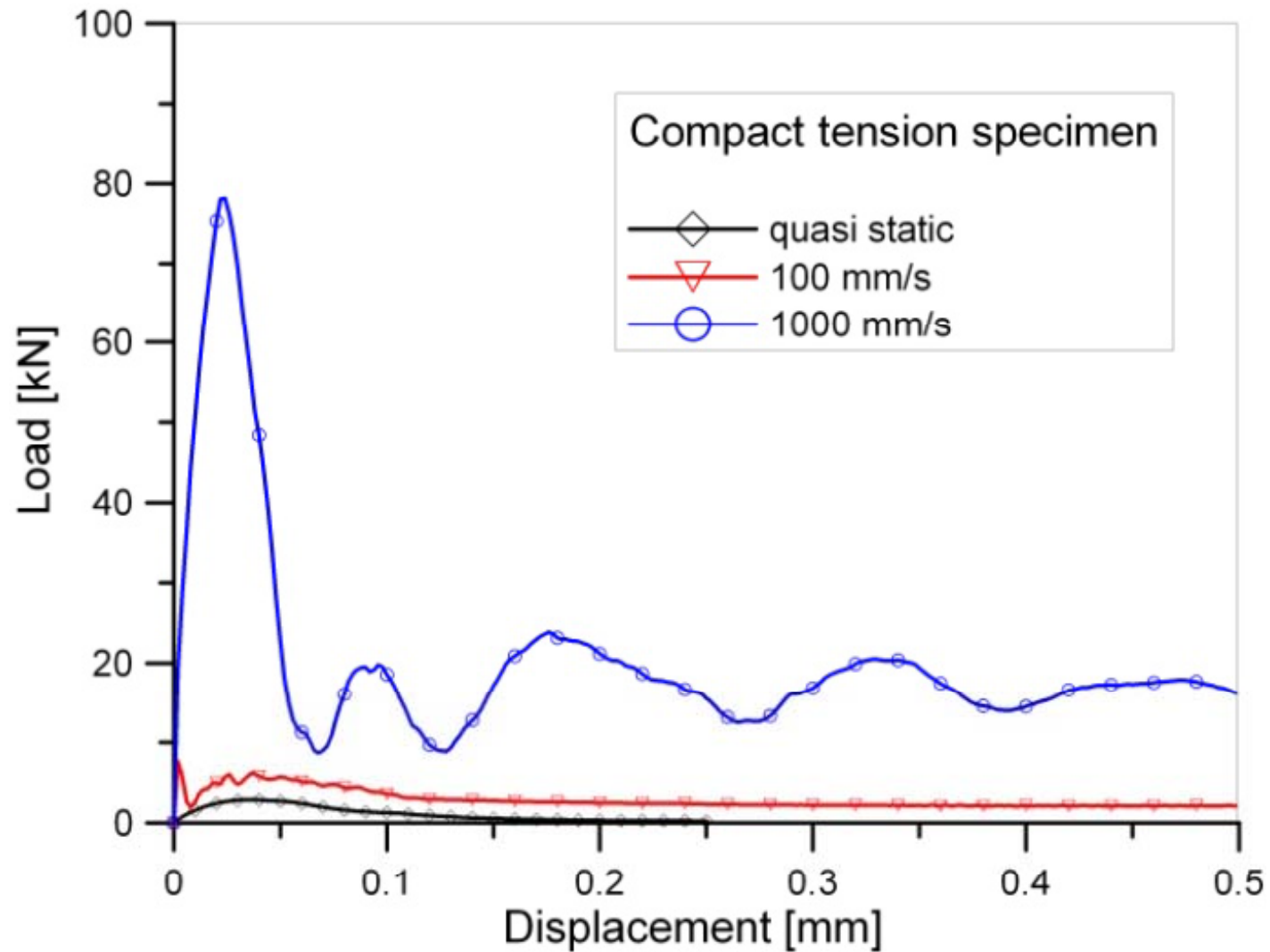
Compact tension specimen



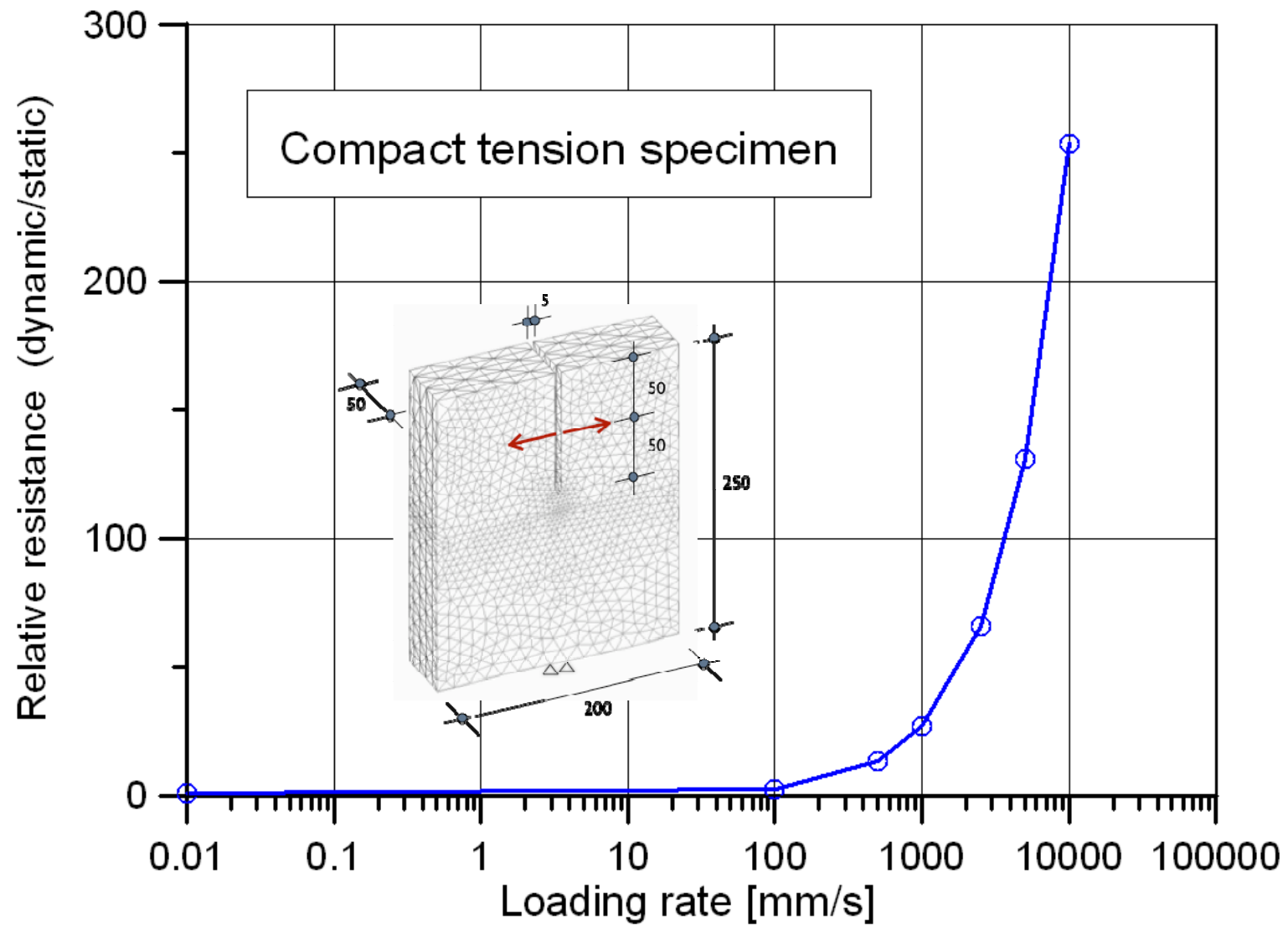
Compact tension specimen



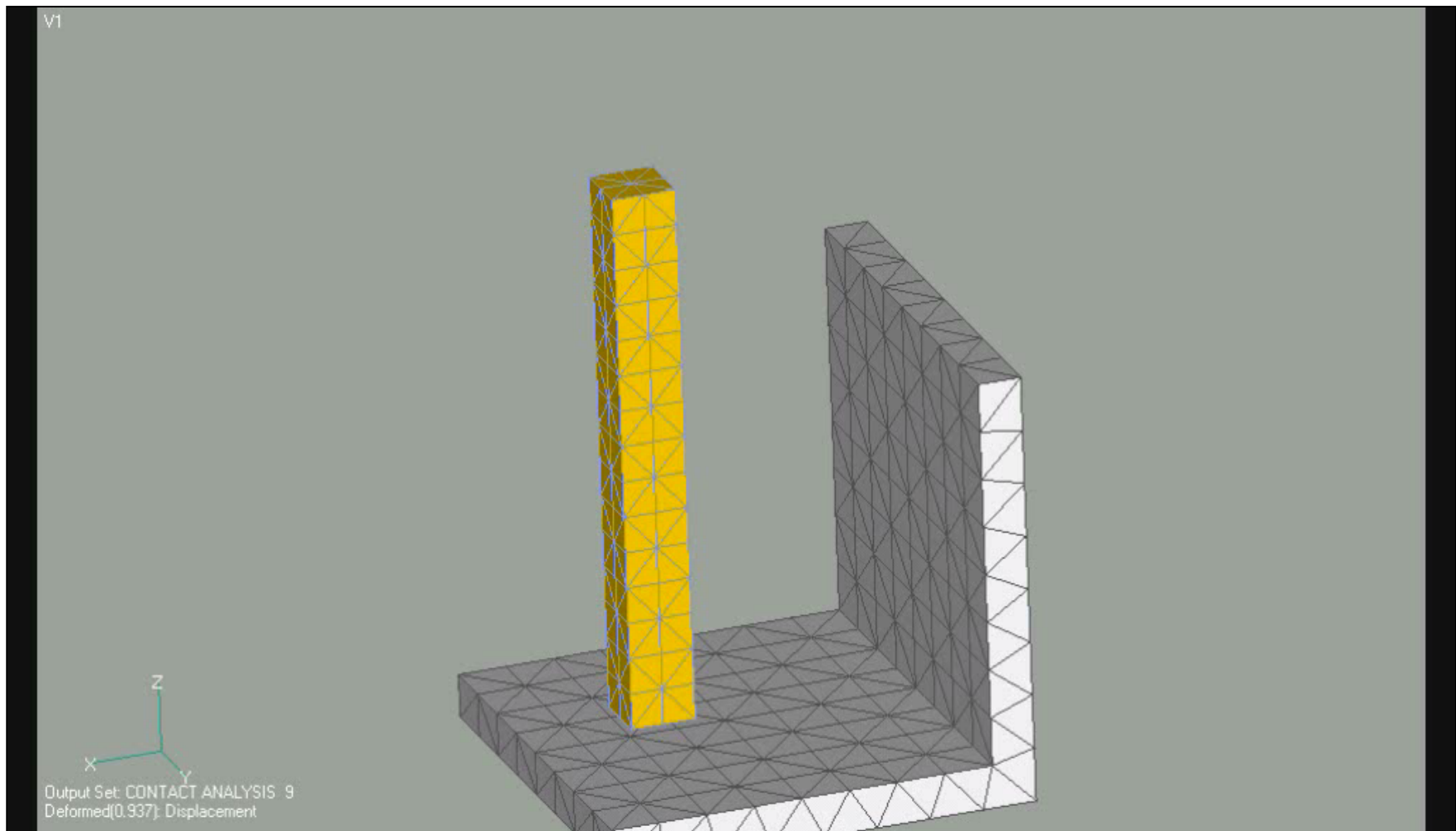
Compact tension specimen



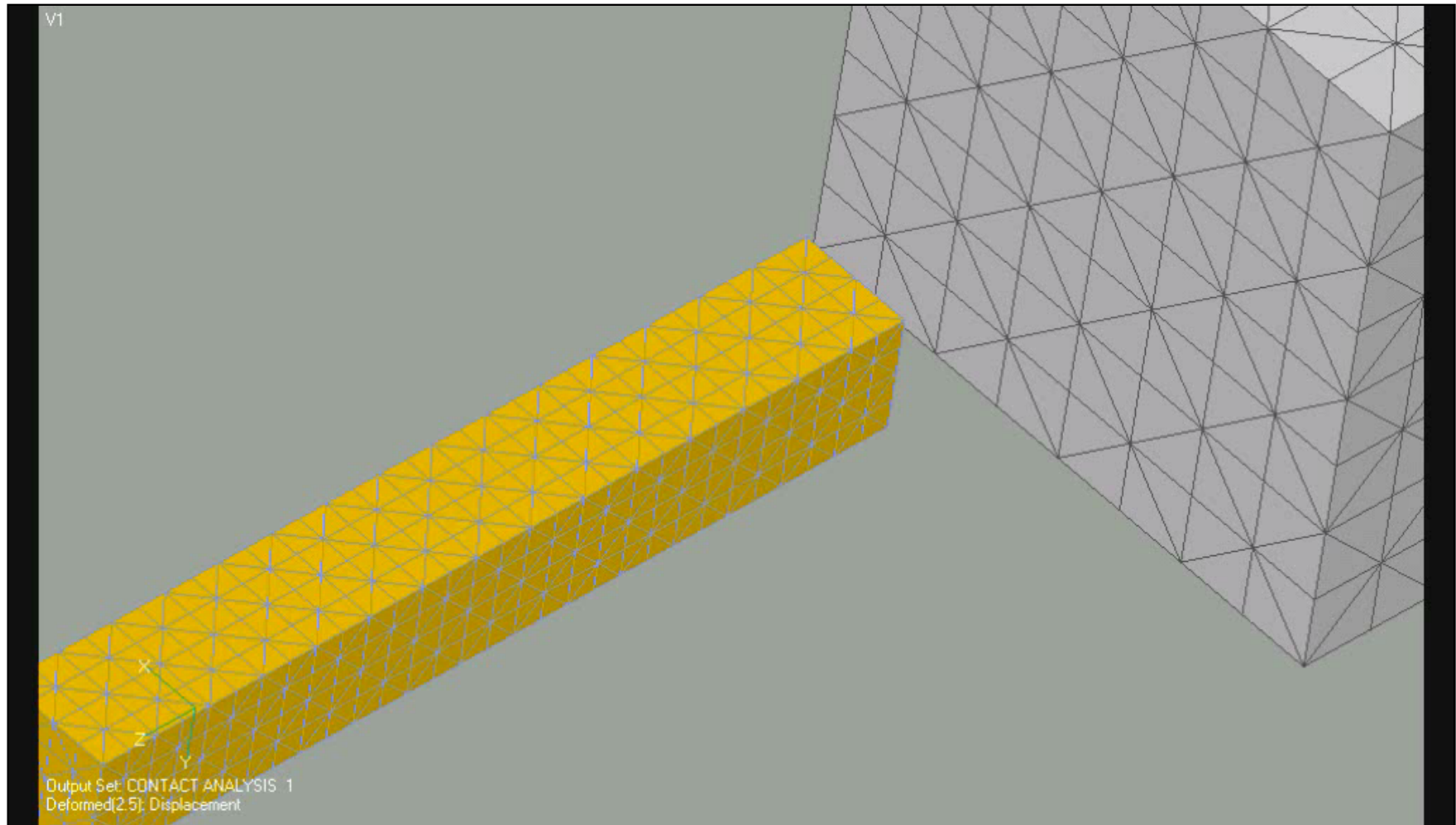
Compact tension specimen



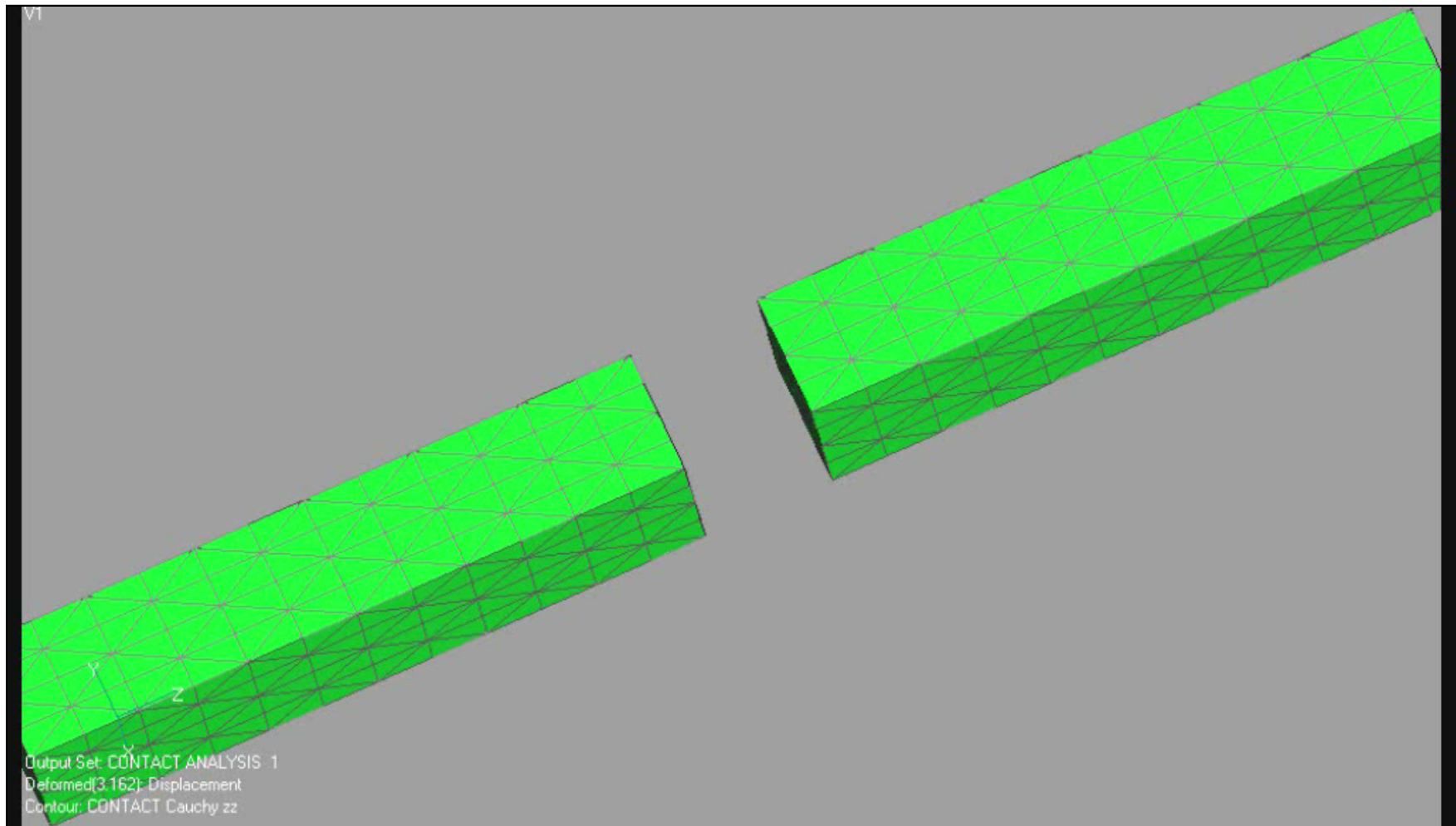
Multi body dynamics & contact



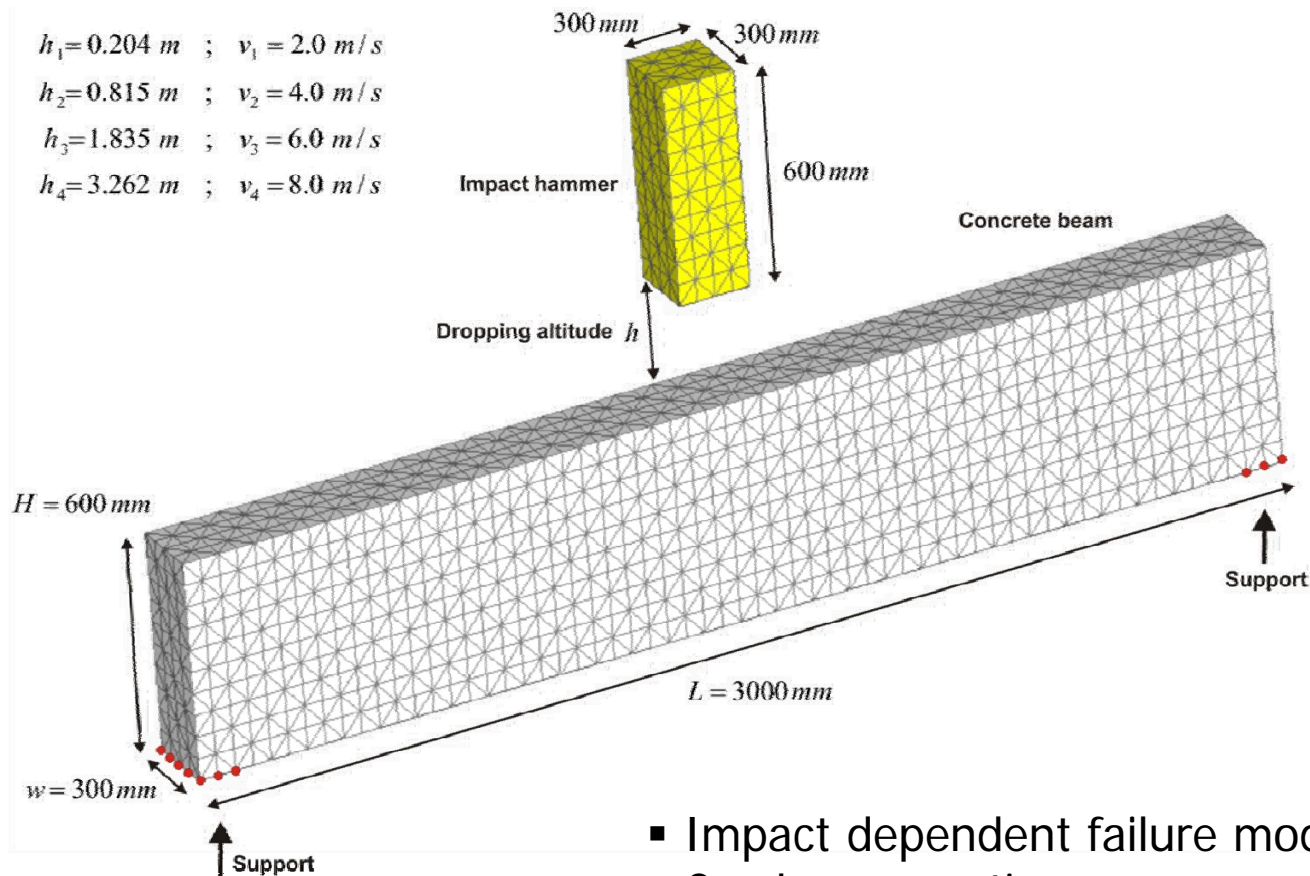
Multi body dynamics & contact



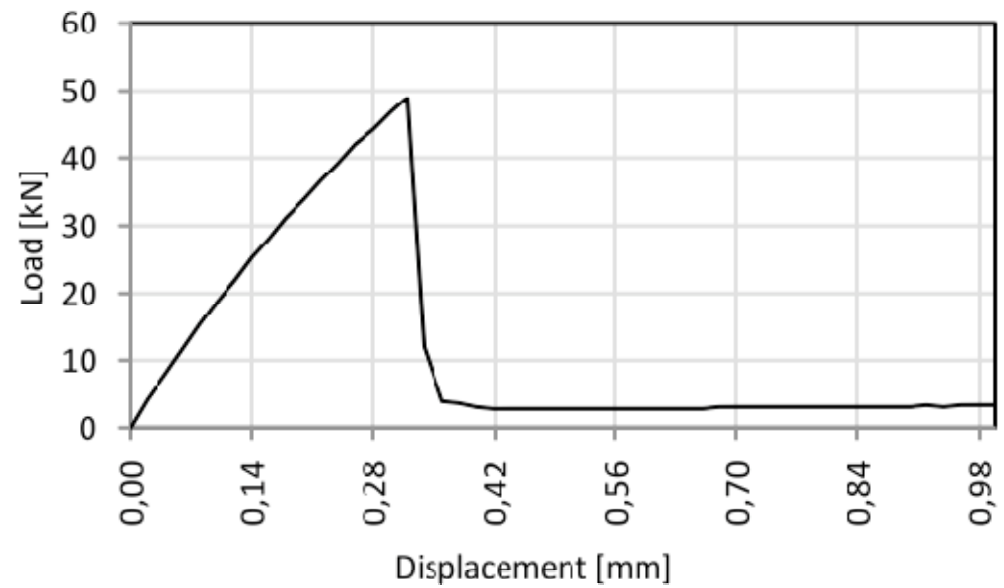
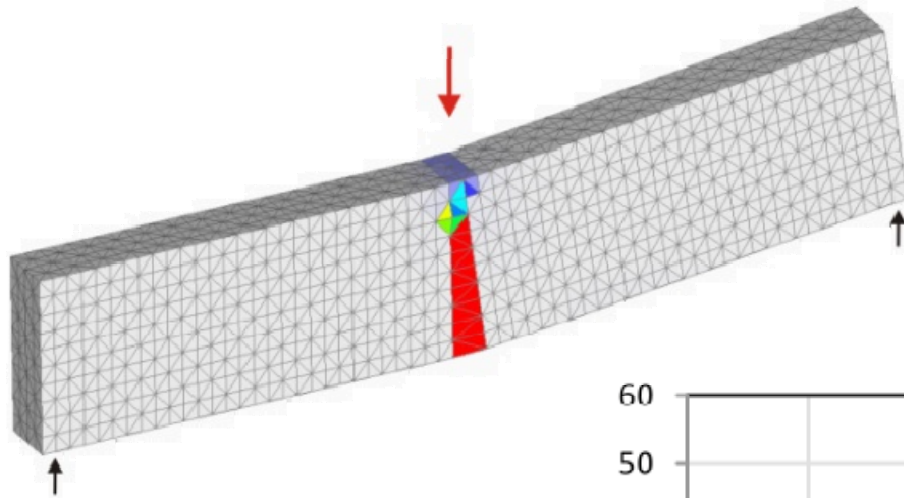
Multi body dynamics & contact



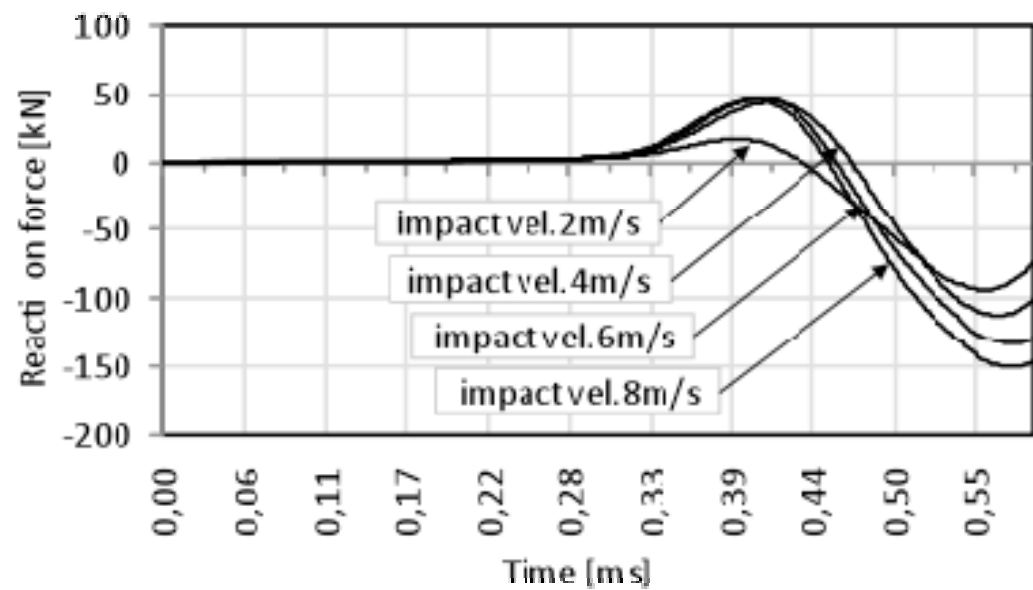
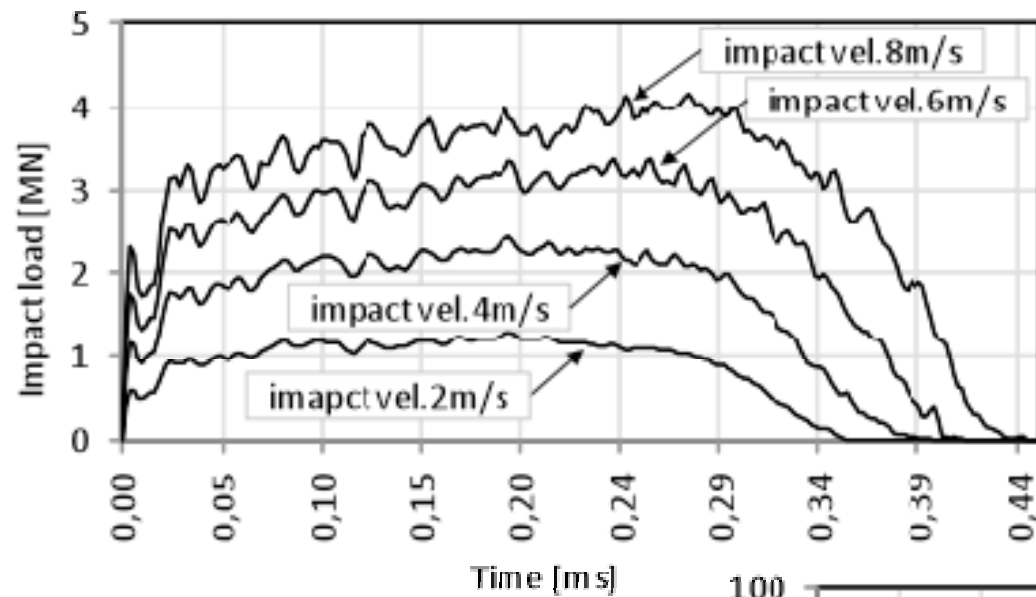
Impact – simply supported plain concrete beam



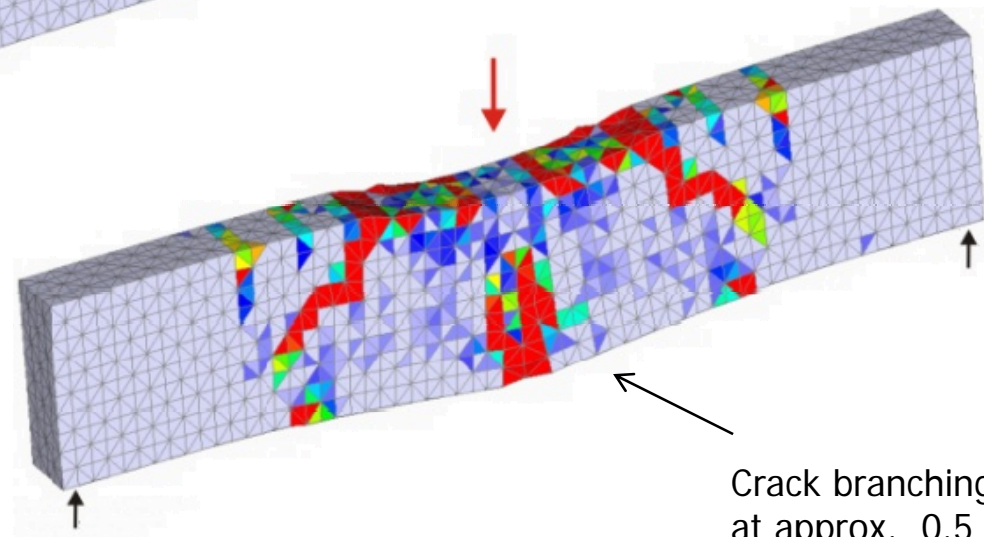
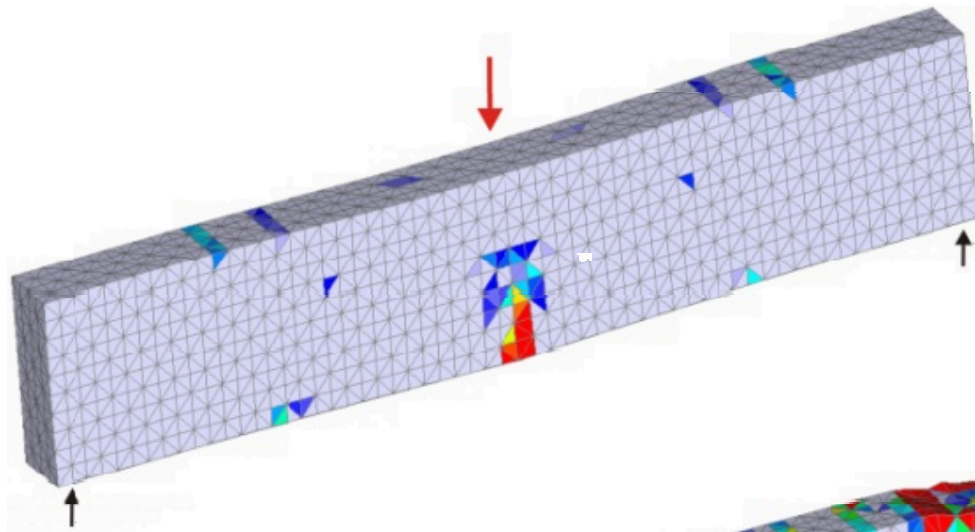
Quasi-static loading



Impact load & reactions

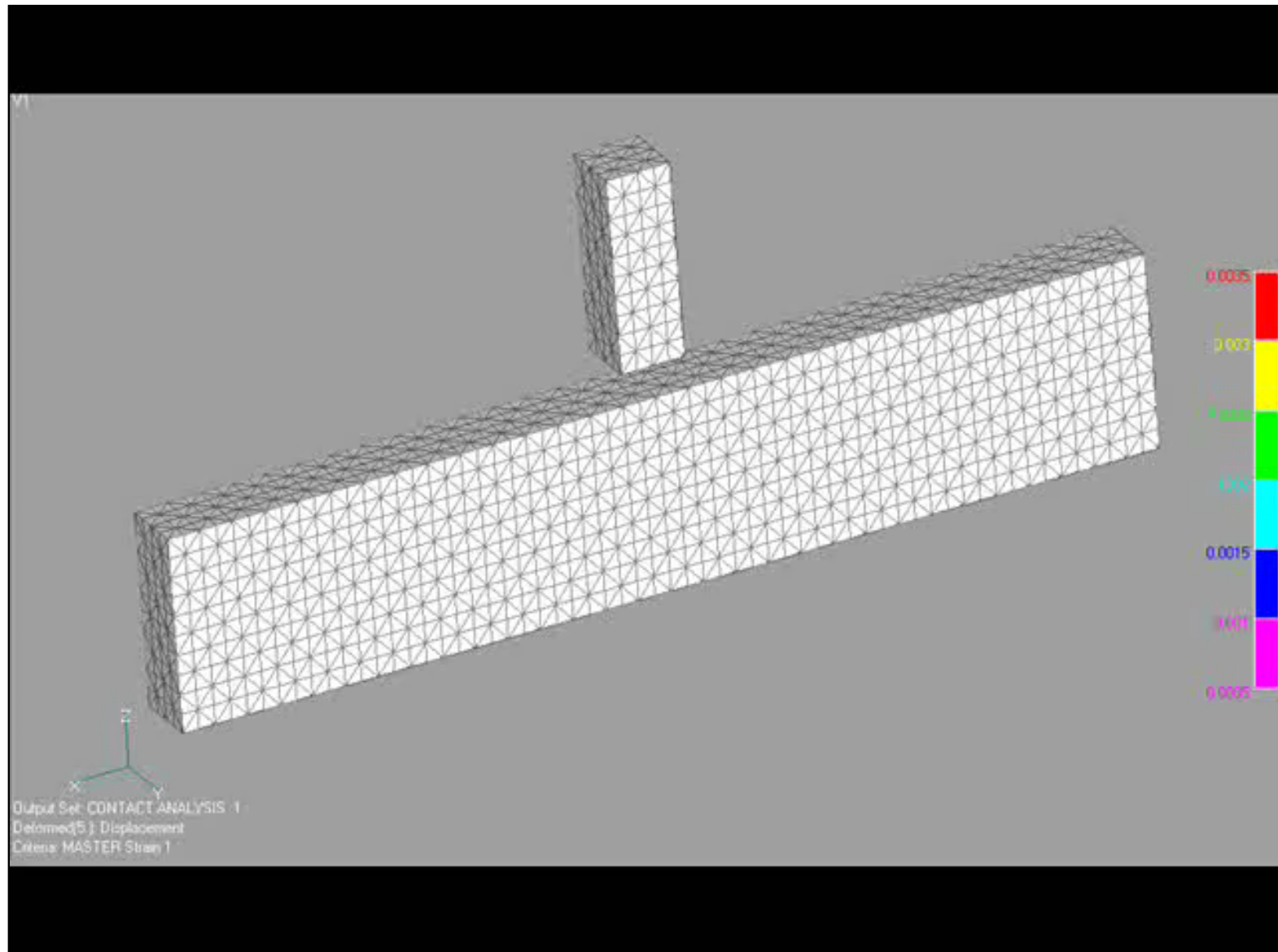


Impact velocity 2 m/s – bending failure

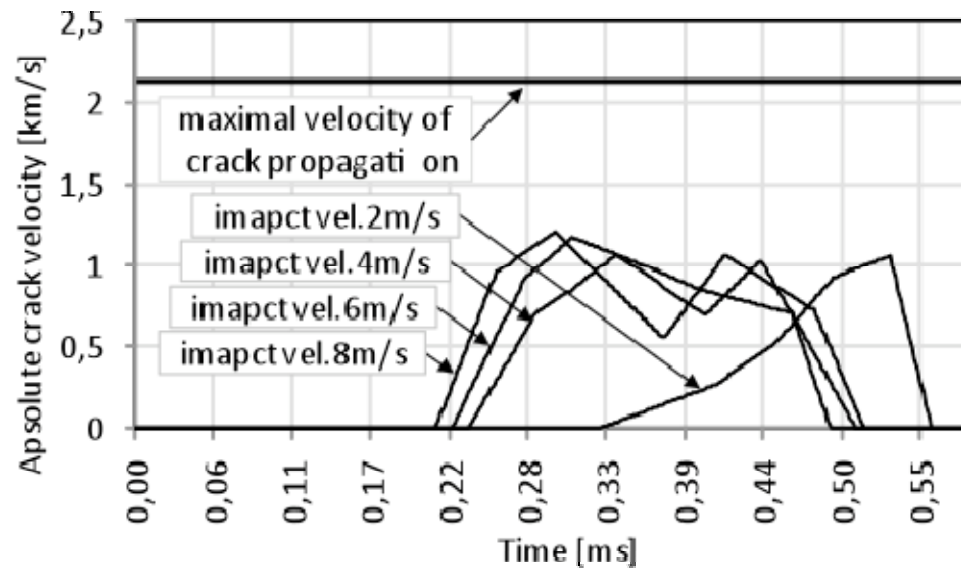
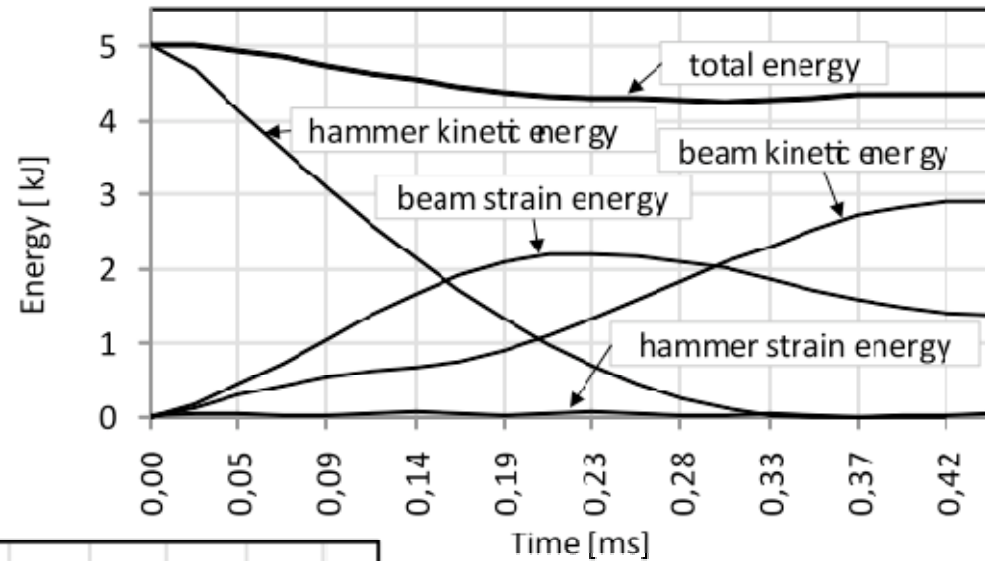
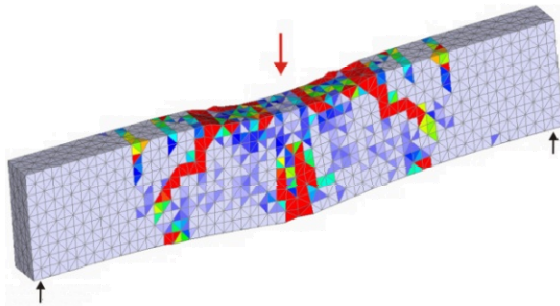


Impact velocity 8 m/s – mixe-mode failure

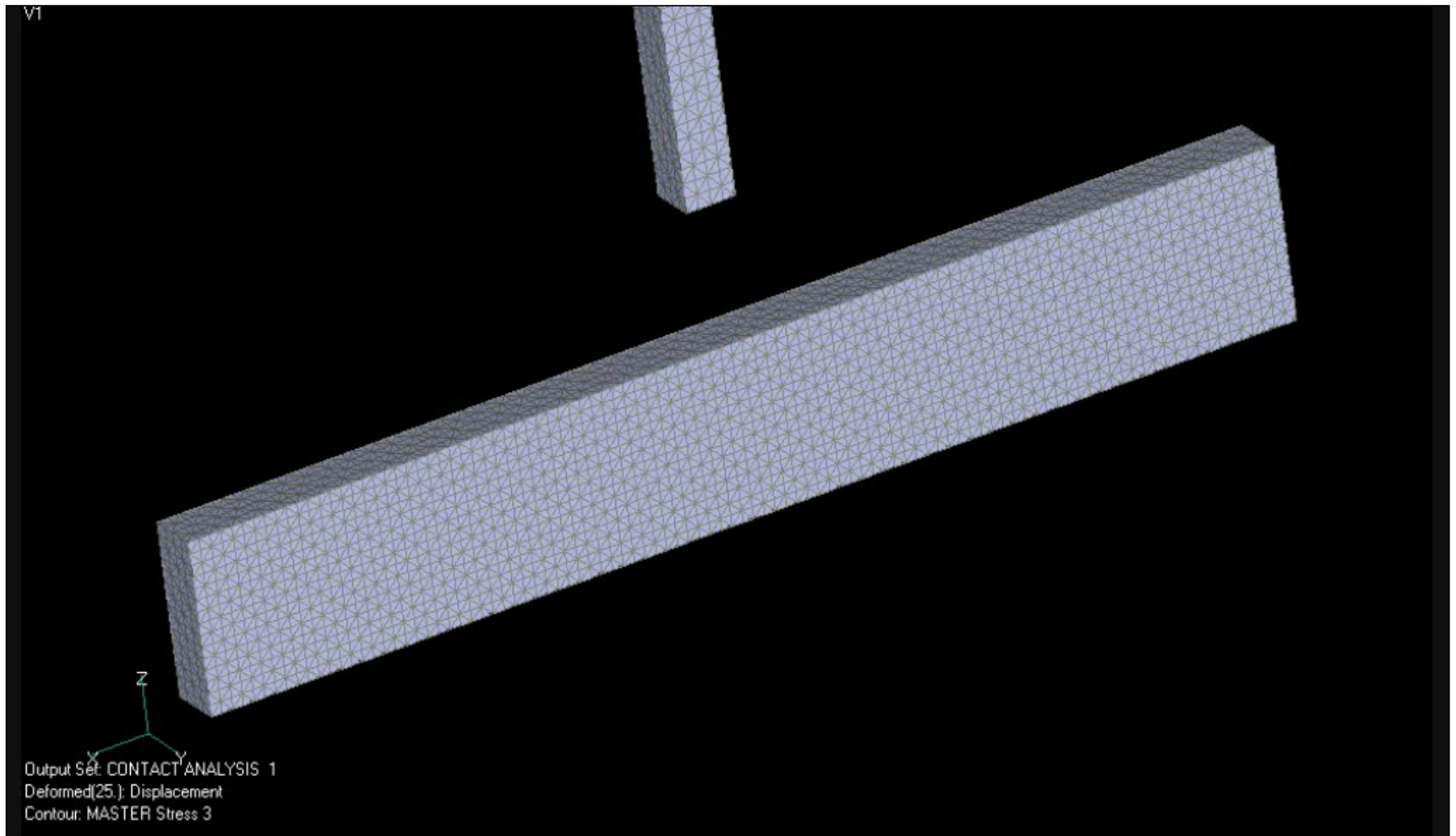
Impact velocity 8 m/s



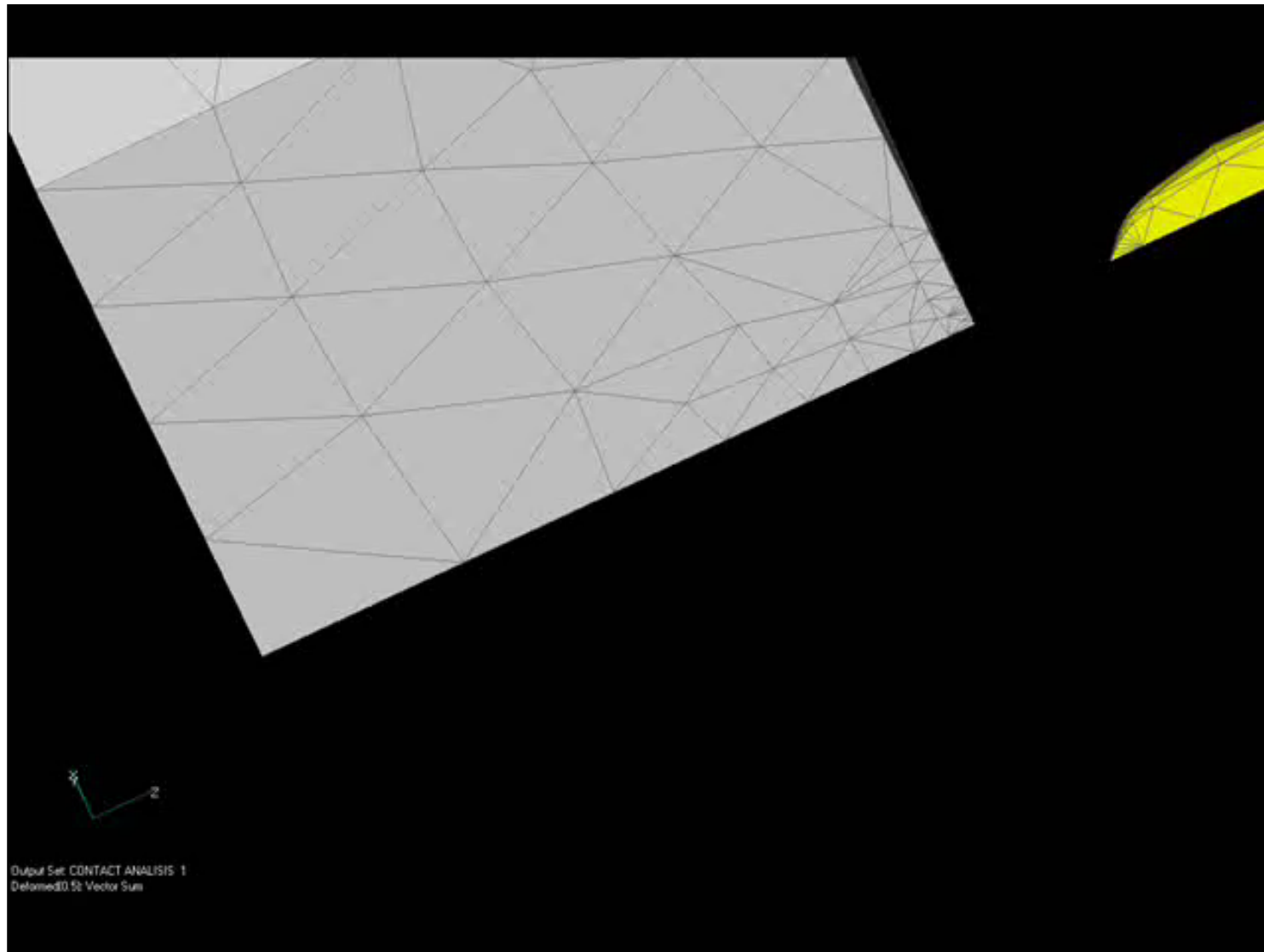
Impact velocity 8 m/s – shear failure



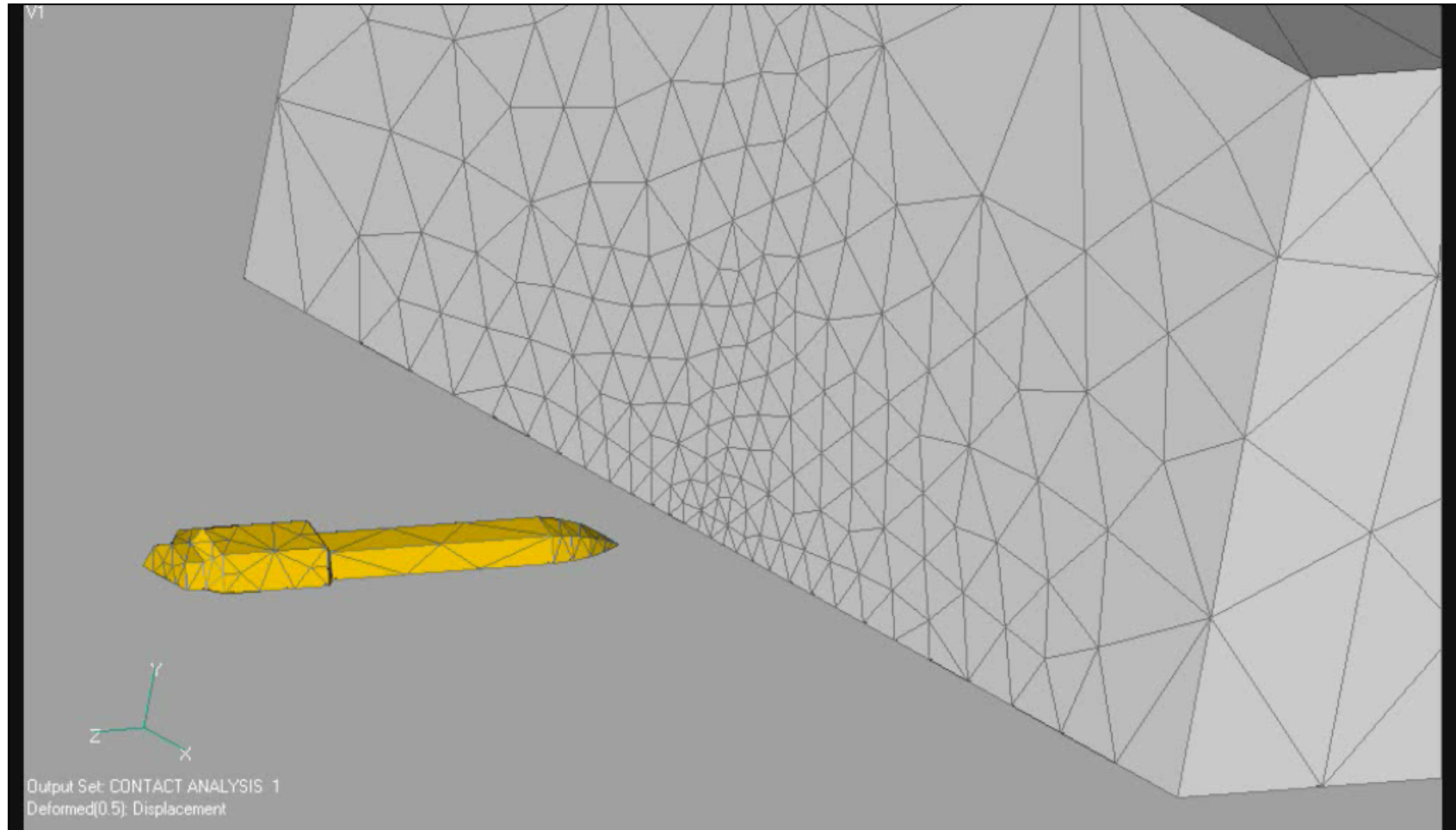
Compressive waves



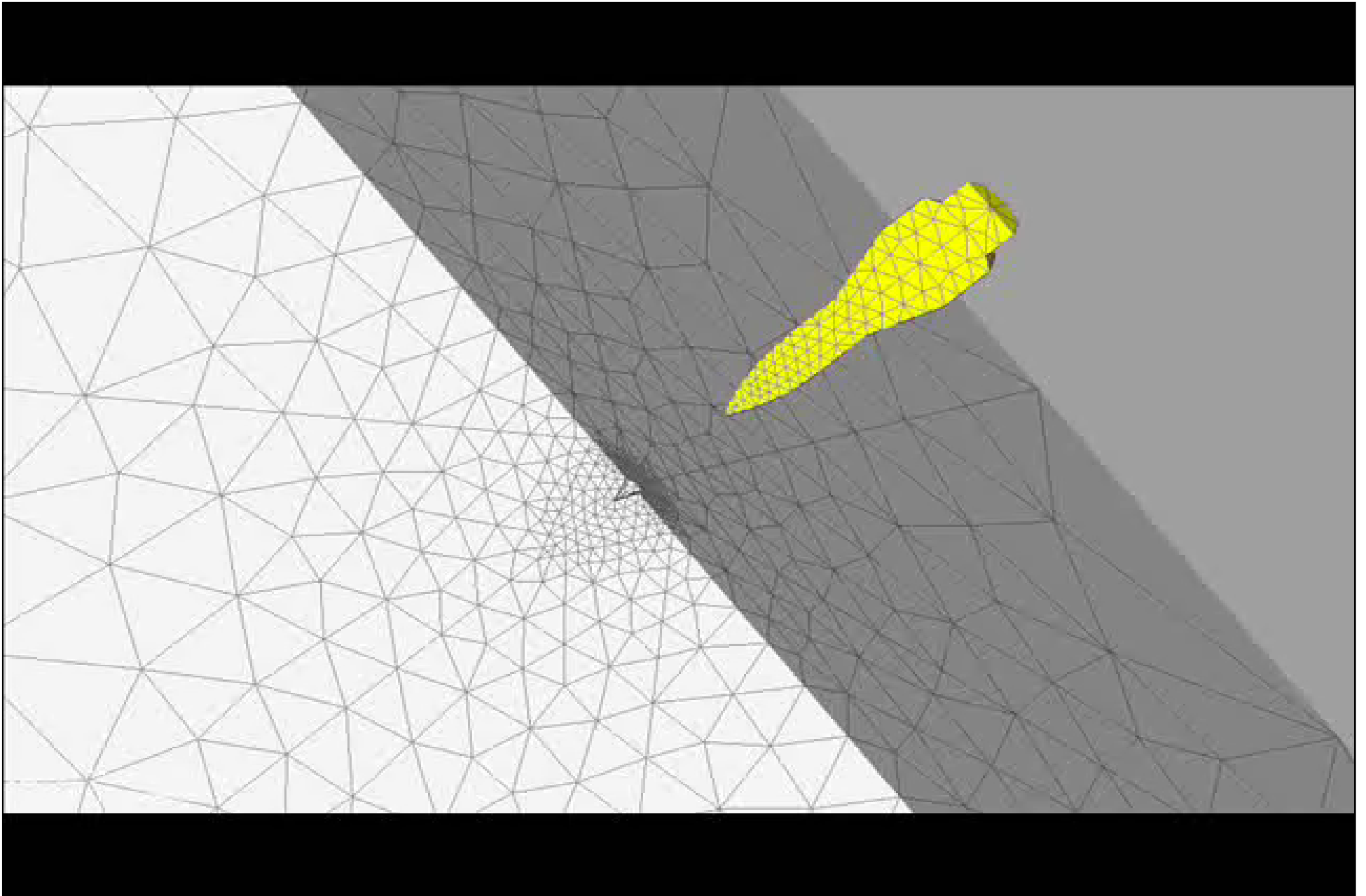
Impact - penetration of nail



Impact - penetration of nail



Impact - penetration of nail



Summary

- With increase of loading rate, resistance and brittleness of the structural response increase and failure mode is rate dependent
- At moderate loading rates the response is controlled by the phenomena at the material micro, however, for higher loading rates inertia forces dominate
- At very high loading rates inertia forces dominate & contact mechanics is important part of the problem to be solved (friction & heat)
- There is a strong influence of the structural size on the loading rate response
- The used rate dependent model is simple and covers a broad range of strain rates